

CALCULO
GRADO EN INGEN. INFORM. DEL SOFTWARE. 12-13
TEMA 2. ACTIVIDADES 2.1 A 2.3

2.1) Calcular las siguientes integrales indefinidas:

a) $\int \frac{1}{(3-x)^4} dx$; b) $\int \frac{1}{\operatorname{tg}(2x)} dx$; c) $\int \frac{1}{2+32x^2} dx$
 d) $\int \operatorname{arctg}(x) dx$; e) $\int \frac{1}{x^2(x-1)} dx$; f) $\int \frac{x^2}{\sqrt{16-4x^2}} dx$

Solución.

a) $\int \frac{1}{(3-x)^4} dx = \frac{1}{3(3-x)^3} + C$

b) $\int \frac{1}{\operatorname{tg}(2x)} dx = \frac{1}{2} \log(\operatorname{sen}(2x)) + C$

c) $\int \frac{1}{2+32x^2} dx = \frac{1}{8} \operatorname{arctg}(4x) + C$

d) $\int \operatorname{arctg}(x) dx = x \cdot \operatorname{arctg}(x) - \int \frac{x}{1+x^2} dx = x \cdot \operatorname{arctg}(x) - \frac{1}{2} \log(1+x^2) + C$

e) $\int \frac{1}{x^2(x-1)} dx = \int \frac{A}{x} dx + \int \frac{B}{x^2} dx + \int \frac{C}{x-1} dx = \int \frac{-1}{x} dx + \int \frac{-1}{x^2} dx + \int \frac{1}{x-1} dx =$
 $= -\log|x| + \frac{1}{x} + \log|x-1| = \log\left|\frac{x-1}{x}\right| + \frac{1}{x} + C$

f) $\int \frac{x^2}{\sqrt{16-4x^2}} dx = \frac{1}{2} \int \frac{x^2}{\sqrt{4-x^2}} dx = (\text{cambio de variable: } x = 2\operatorname{sen}(t))$
 $= \frac{1}{2} \int \frac{4\operatorname{sen}^2(t)}{\sqrt{4-4\operatorname{sen}^2(t)}} 2\cos(t) dt = 2 \int \operatorname{sen}^2(t) dt = \int (1 - \cos(2t)) dt = t - \frac{1}{2} \operatorname{sen}(2t) =$
 $= \operatorname{arcsin}\left(\frac{x}{2}\right) - \frac{x}{2} \sqrt{1 - \frac{x^2}{4}} + C$

2.2) Calcular una primitiva de las funciones

a) $f(x) = \frac{\log(x)}{x^{3/2}}$; b) $f(x) = \frac{x}{(x-1)^2(x+1)}$; c) $f(x) = \frac{x-1}{\sqrt{x+1}}$

Solución.

a)

Aplicamos el método de integración por partes:

$$u = \log(x) \Rightarrow du = \frac{1}{x} dx \quad ; \quad dv = x^{-3/2} dx \Rightarrow v = \frac{x^{-1/2}}{-1/2} = -2x^{-1/2}$$

$$\int \log(x) x^{-3/2} dx = -2x^{-1/2} \log(x) - \int -2x^{-1/2} x^{-1} dx = -2x^{-1/2} \log(x) + 2 \int x^{-3/2} dx =$$

$$= -2x^{-1/2} \log(x) - 4x^{-1/2} + C = -2x^{-1/2} [\log(x) + 2] + C$$

Una primitiva es $-2x^{-1/2} [\log(x) + 2]$

b)

$$\frac{x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} = \frac{1/4}{x-1} + \frac{1/2}{(x-1)^2} + \frac{-1/4}{x+1}$$

Una primitiva es $\frac{1}{4} \log|x-1| - \frac{1}{2} \frac{1}{x-1} - \frac{1}{4} \log|x+1| + C$

c) Una primitiva es $\frac{2(x-5)\sqrt{x+1}}{3}$

2.3) Calcular $\int_{-3}^4 |x^2 - 4| dx$

Solución.

$$|x^2 - 4| = \begin{cases} x^2 - 4 & \text{si } x \geq 2 \text{ ó } x \leq -2 \\ 4 - x^2 & \text{si } -2 \leq x \leq 2 \end{cases}$$

$$\int_{-3}^4 |x^2 - 4| dx = \int_{-3}^{-2} (x^2 - 4) dx + \int_{-2}^2 (4 - x^2) dx + \int_2^4 (x^2 - 4) dx = \frac{7}{3} + \frac{32}{3} + \frac{32}{3} = \frac{71}{3}$$