Tables for Signals and Systems

Geometric and Arithmetic Series

$\sum_{m=0}^{N} r^m = \frac{1-r^{N+1}}{1-r}$, if $r \neq 1$	$\sum_{m=1}^{N} r^m = \frac{r-r^{N+1}}{1-r}$, if $r \neq 1$	$\sum_{m=-N}^{N} r^m = \frac{r^{-N}-r^{N+1}}{1-r}$, if $r \neq 1$
$a_m = a_{m-1} + d \Rightarrow \sum_{m=N_1}^{N_2} a_m = \frac{a_{N_1}+a_{N_2}}{2}$		

Distributions

Definition: $\int_{-\infty}^{\infty} f(x)\delta^{(n)}(x-x_0)dx = (-1)^n f^{(n)}(x_0)$	
$f(x)\delta(x-x_0) = f(x_0)\delta(x-x_0)$	$f(x)\delta'(x-x_0) = f(x_0)\delta'(x-x_0) - f'(x_0)\delta(x-x_0)$
$x\delta(x) = 0$	$x\delta'(x) = -\delta(x)$
$f(x) * \delta(x-x_0) = f(x-x_0)$	$f(x) * \delta'(x-x_0) = f'(x-x_0)$
$\delta(ax-x_0) = \frac{1}{\|a\|}\delta(x-\frac{x_0}{a})$	$\delta'(ax-x_0) = \frac{1}{\|a\|}\frac{1}{a}\delta'(x-\frac{x_0}{a})$
$\delta(x) = \delta(-x)$	$\delta'(x) = -\delta'(-x)$

Fourier Series - Continuous-variable

The fundamental period is defined as $\xi_0 = \frac{2\pi}{X_0}$

Properties	
$\alpha f_0(x) + \beta g_0(x)$	$\alpha a(m) + \beta b(m)$
$f_0(x-x_o)$	$a(m)e^{-jm\xi_0 x_0}$
$f_0(x)e^{jm_0\xi_o}$	$a(m-m_0)$
$f_0^*(x)$	$a^*(-m)$
$f_0(-x)$	$a(-m)$
$f_0(\alpha x)$, if $\alpha > 0$	$a(m)$ (period $X_0' = \frac{X_0}{\alpha}$)
$f_0(x) * g_0(x)$	$X_0 a(m)b(m)$
$f_0(x)g_0(x)$	$a(m) * b(m)$
$\frac{d}{dx}f_0(x)$	$jm\xi_0 a(m)$
$\frac{d^n}{dx^n}f_0(x)$	$(jm\xi_0)^n a(m)$
$\int f_0(x)dx$ [1]	$\frac{1}{jm\xi_0}a(m)$
$\langle f_0(x), g_0(x)\rangle = X_0 \langle a(m), b(m)\rangle$	
$f_0(x) = f_0^*(x)$ (real)	$a(m) = a^*(-m)$
$f_0(x) = f_0(-x)$ (even)	$a(m) = a(-m)$
$f_0(x) = -f_0(-x)$ (odd)	$a(m) = -a(-m)$

Transform pairs	
$f_0(x)$	$a(m)$
$e^{jk\xi_0 x}$	$\delta(m-k)$
$f_0(x) = \sum_{m=-\infty}^{\infty} a(m)e^{jm\xi_0 x}$	$a(m) = \frac{1}{X_0}\int_{\langle X_0\rangle} f_0(x)e^{-jm\xi_0 x}dx$
K	$K\delta(m)$
$\cos(k\xi_0 x)$	$\frac{1}{2}\left[\delta(m-k)+\delta(m+k)\right]$
$\sin(k\xi_0 x)$	$\frac{1}{2j}\left[\delta(m-k)-\delta(m+k)\right]$
$\Pi_{0,\Delta}(x) = \sum_{k=-\infty}^{\infty} \Pi\left(\frac{x-kX_0}{\Delta}\right)$	$\frac{\Delta}{X_0}\text{sinc}\left(m\pi\frac{\Delta}{X_0}\right)$
$\Lambda_{0,\Delta}(x) = \sum_{k=-\infty}^{\infty} \Lambda\left(\frac{x-kX_0}{\Delta}\right)$	$\frac{\Delta}{2X_0}\text{sinc}^2\left(m\pi\frac{\Delta}{2X_0}\right)$
$\delta_0(x) = \sum_{k=-\infty}^{\infty} \delta(x-kX_0)$	$\frac{1}{X_0}$
$\sum_{k=-\infty}^{\infty} \delta(x-kX_0) = \frac{1}{X_0}\sum_{m=-\infty}^{\infty} e^{jm\xi_0 x}$ (Poisson summation formula)	

[1] only if $\langle f_0(x)\rangle = a(0) = 0$

Fourier Transform - Continuous-variable

Properties	
$\alpha f(x)+\beta g(x)$	$\alpha F(\xi)+\beta G(\xi)$
$f(x-x_o)$	$F(\xi)e^{-j\xi x_0}$
$f(x)e^{j\xi_o x}$	$F(\xi-\xi_0)$
$f^*(x)$	$F^*(-\xi)$
$f(-x)$	$F(-\xi)$
$f(ax)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\xi}{a}\right)$
$f(x)*g(x)$	$F(\xi)G(\xi)$
$f(x)g(x)$	$\frac{1}{2\pi}F(\xi)*G(\xi)$
$\frac{d}{dx}f(x)$	$j\xi F(\xi)$
$\frac{d^n}{dx^n}f(x)$	$(j\xi)^n F(\xi)$
$x^n f(x)$	$j^n\frac{d^n F(\xi)}{d\xi^n}$
$\int_{-\infty}^{x} f(y)dy$	$\frac{1}{j\xi}F(\xi)+\pi F(0)\delta(\xi)$
$F(x)$	$2\pi f(-\xi)$
$\langle f(x),g(x)\rangle=\frac{1}{2\pi}\langle F(\xi),G(\xi)\rangle$	
$f(x)=f^*(x)$ real	$F(\xi)=F^*(-\xi)$
$f(x)=f(-x)$ even	$F(\xi)=F(-\xi)$
$f(x)=-f(-x)$ odd	$F(\xi)=-F(-\xi)$

Transform pairs	
$f(x)$	$F(\xi)$
$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\xi)e^{j\xi x}d\xi$	$F(\xi)=\int_{-\infty}^{\infty}f(x)e^{-j\xi x}dx$
$\delta(x)$	1
$\delta(x-x_0)$	$e^{-j\xi x_0}$
$u(x)$	$\pi\delta(\xi)+\frac{1}{j\xi}$
$\Pi\left(\frac{x}{\Delta}\right)$	$\Delta\text{sinc}\left(\frac{\Delta}{2}\xi\right)$
$\frac{\Delta}{2\pi}\text{sinc}\left(\frac{\Delta}{2}x\right)$	$\Pi(\frac{\xi}{\Delta})$
$\Lambda\left(\frac{x}{\Delta}\right)$	$\frac{\Delta}{2}\text{sinc}^2\left(\frac{\Delta}{4}\xi\right)$
$e^{-\alpha x}u(x)$, $\Re\{\alpha\}>0$	$\frac{1}{\alpha+j\xi}$
$xe^{-\alpha x}u(x)$, $\Re\{\alpha\}>0$	$\frac{1}{(\alpha+j\xi)^2}$
$\frac{x^{n-1}}{(n-1)!}e^{-\alpha x}u(x)$, $\Re\{\alpha\}>0$	$\frac{1}{(\alpha+j\xi)^n}$
$e^{j\xi_0 x}$	$2\pi\delta(\xi-\xi_0)$
K	$2\pi K\delta(\xi)$
$\cos(\xi_0 x)$	$\pi[\delta(\xi-\xi_0)+\delta(\xi+\xi_0)]$
$\sin(\xi_0 x)$	$\frac{\pi}{j}[\delta(\xi-\xi_0)-\delta(\xi+\xi_0)]$
$\sum_{m=-\infty}^{\infty}a(m)e^{jm\xi_0 x}$	$2\pi\sum_{m=-\infty}^{\infty}a(m)\delta(\xi-m\xi_0)$
$\sum_{k=-\infty}^{\infty}\delta(x-kX_0)$	$\frac{2\pi}{X_0}\sum_{m=-\infty}^{\infty}\delta(\xi-m\xi_0)$
$\Pi_{0,\Delta}(x)=\sum_{k=-\infty}^{\infty}\Pi\left(\frac{x-kX_0}{\Delta}\right)$	$\frac{2\pi\Delta}{X_0}\sum_{m=-\infty}^{\infty}\text{sinc}\left(m\pi\frac{\Delta}{X_0}\right)\delta(\xi-m\xi_0)$

Fourier Series - Discrete-variable

The fundamental period is defined as $\Omega_0=\frac{2\pi}{N_0}$.

Properties	
$\alpha x_0(n)+\beta y_0(n)$	$\alpha a(m)+\beta b(m)$
$x_0(n-n_0)$	$a(m)e^{-jm\Omega_0 n_0}$
$x_0(n)e^{jM\Omega_0 n}$	$a(m-M)$
$x_0^*(n)$	$a^*(-m)$
$x_0(-n)$	$a(-m)$
$\begin{cases}x_0(n/p) & n \text{ mult. } p\\ 0 & n \text{ not mult. } p\end{cases}$	$\frac{1}{p}a(m)$
$x_0(n)*y_0(n)$	$N_0a(m)b(m)$
$x_0(n)y_0(n)$	$\sum_{k=\langle N_0\rangle}a(k)b(k-m)$
$x_0(n)-x_0(n-1)$	$\left(1-e^{-jm\Omega_0}\right)a(m)$
$\sum_{k=-\infty}^{n}x_0(k)$, if $a(0)=0$	$\frac{1}{(1-e^{-jm\Omega_0})}a(m)$
$\langle x_o(n),y_o(n)\rangle=N_o\langle a(m),b(m)\rangle$	
$x_0(n)=x_0^*(n)$ real	$a(m)=a^*(-m)$
$x_0(n)=x_0(-n)$ even	$a(m)=a(-m)$
$x_0(n)=-x_0(-n)$ odd	$a(m)=-a(-m)$

Transform pairs	
$x_0(n)$	$a(m)$
$e^{jp\Omega_0 n}$	$\sum_{k=-\infty}^{\infty}\delta(m-p-kN_0)$
$x_0(n)=\sum_{m=\langle N_0\rangle}a(m)e^{jm\Omega_0 n}$	$a(m)=\frac{1}{N_0}\sum_{n=\langle N_0\rangle}x_0(n)e^{-jm\Omega_0 n}$
$\cos(p\Omega_0 n)$	$\frac{1}{2}\sum_{k=-\infty}^{\infty}\delta(m-p-kN_0)+\frac{1}{2}\sum_{k=-\infty}^{\infty}\delta(m+p-kN_0)$
$\sin(p\Omega_0 n)$	$\frac{1}{2j}\sum_{k=-\infty}^{\infty}\delta(m-p-kN_0)-\frac{1}{2j}\sum_{k=-\infty}^{\infty}\delta(m+p-kN_0)$
K	$\sum_{k=-\infty}^{\infty}K\delta(m-kN_0)$
$\delta_0(n)=\sum_{k=-\infty}^{\infty}\delta(n-kN_0)$	$\frac{1}{N_0}$
$\sum_{k=-\infty}^{\infty}\Pi_{2N+1}(n-kN_0)$	$\frac{1}{N_0}\frac{\sin\left(m\Omega_0\left(N+\frac{1}{2}\right)\right)}{\sin\left(\frac{m\Omega_0}{2}\right)}$
$a(n)$	$\frac{1}{N_0}x_0(-m)$

Fourier Transform - Discrete-variable

Properties	
$X(\Omega) = X(\Omega + 2\pi)$	
$\alpha x(n) + \beta y(n)$	$\alpha X(\Omega) + \beta Y(\Omega)$
$x(n - n_o)$	$X(\Omega) e^{-j\Omega n_o}$
$x(n) e^{j\Omega_o n}$	$X(\Omega - \Omega_o)$
$x^*(n)$	$X^*(-\Omega)$
$x(-n)$	$X(-\Omega)$
$f_0(x) \longleftrightarrow a(m)$ DSF-VC $x(n) = a(-n) \longleftrightarrow X(\Omega) = f_0(\Omega)$ TF-VD with $X_0 = 2\pi$	
$x(n/p)$	$X(p\Omega)$
$x(n) * y(n)$	$X(\Omega) Y(\Omega)$
$x(n) y(n)$	$\frac{1}{2\pi} X(\Omega) * Y(\Omega)$
$x(n) - x(n-1)$	$\left(1 - e^{-j\Omega}\right) X(\Omega)$
$\sum_{k=-\infty}^{n} x(k)$	$\frac{1}{\left(1 - e^{-j\Omega}\right)} X(\Omega) +$ $+\pi \sum_{k=-\infty}^{\infty} \delta\left(\Omega - 2\pi k\right)$
$n x(n)$	$j \frac{dX(\Omega)}{d\Omega}$
$\langle x(n), y(n) \rangle = \frac{1}{2\pi} \langle X(\Omega), Y(\Omega) \rangle$ (Parseval)	
$x(n) = x^*(n)$ (real)	$X(\Omega) = X^*(-\Omega)$
$x(n) = x(-n)$ (even)	$X(\Omega) = X(-\Omega)$
$x(n) = -x(-n)$ (odd)	$X(\Omega) = -X(-\Omega)$

Transform pairs	
$x(n)$	$X(\Omega)$
$x(n) = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(\Omega) e^{j\Omega n} d\Omega$	$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$
$\delta(n)$	1
$\delta(n - n_0)$	$e^{-j\Omega n_0}$
$u(n)$	$\frac{1}{1 - e^{-j\Omega}} + \pi \sum_{k=-\infty}^{\infty} \delta\left(\Omega - 2\pi k\right)$
$e^{j\Omega_0 n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta\left(\Omega - \Omega_0 - 2\pi k\right)$
$\Pi_{2N+1}(n)$	$\frac{\sin\left(\left(N + \frac{1}{2}\right)\Omega\right)}{\sin\left(\frac{\Omega}{2}\right)}$
$a^n u(n)$, $\lvert a \rvert < 1$	$\frac{1}{1 - a e^{-j\Omega}}$
$(n+1) a^n u(n)$, $\lvert a \rvert < 1$	$\frac{1}{(1 - a e^{-j\Omega})^2}$
$\frac{(n+p-1)!}{n!(p-1)!} a^n u(n)$, $\lvert a \rvert < 1$	$\frac{1}{(1 - a e^{-j\Omega})^p}$
K	$2\pi K \sum_{k=-\infty}^{\infty} \delta\left(\Omega - 2\pi k\right)$
$\cos\left(\Omega_0 n\right)$	$\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi k)$ $+\pi \sum_{k=-\infty}^{\infty} \delta(\Omega + \Omega_0 - 2\pi k)$
$\sin\left(\Omega_0 n\right)$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi k)$ $-\frac{\pi}{j} \sum_{k=-\infty}^{\infty} \delta(\Omega + \Omega_0 - 2\pi k)$
$\sum_{m=-\infty}^{\infty} a(m) e^{jm\Omega_0 n}$	$2\pi \sum_{m=-\infty}^{\infty} a(m) \delta\left(\Omega - m\Omega_0\right)$
$\sum_{k=-\infty}^{\infty} \delta\left(n - kN_0\right)$	$\frac{2\pi}{N_0} \sum_{m=-\infty}^{\infty} \delta\left(\Omega - m\Omega_0\right)$
$\sum_{k=-\infty}^{\infty} \Pi_{2N+1}\left(n - kN_0\right)$	$2\pi \sum_{m=-\infty}^{\infty} a(m) \delta\left(\Omega - m\Omega_0\right)$

Laplace Transform

Properties		
$\alpha f(x) + \beta g(x)$	$\alpha F(s) + \beta G(s)$	At least $R_F \cap R_G$
$f(x - x_o)$	$F(s) e^{-s x_o}$	R_F
$f(x) e^{s_o x}$	$F(s - s_o)$	R_F shifted
$f^*(x)$	$F^*(s^*)$	R_F
$f(-x)$	$F(-s)$	R_F reversed
$f(ax)$	$\frac{1}{\lvert a \rvert} F(\frac{s}{a})$	R_F scaled
$f(x) * g(x)$	$F(s) G(s)$	At least $R_F \cap R_G$
$\frac{df(x)}{dx}$	$sF(s)$	At least R_F
$\frac{d^n f(x)}{dx^n}$	$s^n F(s)$	Al least R_F
$\int_{-\infty}^{x} f(y) dy$	$\frac{1}{s} F(s)$	At least $R_F \cap \Re\{s\} > 0$
$-x f(x)$	$\frac{dF(s)}{ds}$	R_F

Transform pairs		
$f(x)$	$F(s)$	ROC
$f(x) = \frac{1}{2\pi j}\int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s)e^{sx}ds$	$F(s) = \int_{-\infty}^{\infty} f(x)e^{-sx}dx$	-
$\delta(x)$	1	$\forall s$
$\delta(x - x_o)$	e^{-sx_o}	$\forall s$
$\delta^{(n)}(x)$	s^n	$\forall s$
$\pm u(\pm x)$	$\frac{1}{s}$	$\Re\{s\} \gtrless 0$
$xu(x)$	$\frac{1}{s^2}$	$\Re\{s\} > 0$
$\pm\frac{x^{n-1}}{(n-1)!}u(\pm x)$	$\frac{1}{s^n}$	$\Re\{s\} \gtrless 0$
$\pm e^{-\alpha x}u(\pm x)$	$\frac{1}{s+\alpha}$	$\Re\{s\} \gtrless -\alpha$
$\pm\frac{x^{n-1}}{(n-1)!}e^{-\alpha x}u(\pm x)$	$\frac{1}{(s+\alpha)^n}$	$\Re\{s\} \gtrless -\alpha$
$\cos(\xi_0 x)u(x)$	$\frac{s}{s^2+\xi_0^2}$	$\Re\{s\} > 0$
$\sin(\xi_0 x)u(x)$	$\frac{\xi_0}{s^2+\xi_0^2}$	$\Re\{s\} > 0$
$e^{-\alpha x}\cos(\xi_0 x)u(x)$	$\frac{s+\alpha}{(s+\alpha)^2+\xi_0^2}$	$\Re\{s\} > -\alpha$
$e^{-\alpha x}\sin(\xi_0 x)u(x)$	$\frac{\xi_0}{(s+\alpha)^2+\xi_0^2}$	$\Re\{s\} > -\alpha$

It is assumed that $\alpha \in \mathbb{R}$.

z-Transform

Properties		
$\alpha x(n) + \beta y(n)$	$\alpha X(z) + \beta Y(z)$	At least $R_X \cap R_Y$
$x(n - n_o)$	$X(z)z^{-n_0}$	R_X (origin?)
$x(n)e^{j\Omega_0 n}$	$X(ze^{-j\Omega_0})$	R_X
$x(n)z_0^n$	$X(\frac{z}{z_0})$	$\|z_0\|R_X$
$x(n)a^n$	$X(\frac{z}{a})$	$\|a\|R_X$
$x^*(n)$	$X^*(z^*)$	R_X
$x(-n)$	$X(1/z)$	$1/R_X$
$x(n/p)$	$X(z^p)$	$[R_X]^{\frac{1}{p}}$
$x(n) * y(n)$	$X(z)Y(z)$	At least $R_X \cap R_Y$
$x(n) - x(n-1)$	$\left(1 - z^{-1}\right)X(z)$	At least R_X
$\sum_{p=-\infty}^{n} x(p)$	$\frac{1}{1-z^{-1}}X(z)$	At least $R_X \cap \|z\| > 1$
$nx(n)$	$-\frac{dX(z)}{dz}$	R_X

Transform pairs		
$x(n)$	$X(z)$	ROC
$x(n) = \frac{1}{2\pi j}\oint_{\|z\|} X(z)z^{n-1}dz$	$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$	-
$\delta(n)$	1	$\forall z$
$\delta(n - n_o)$	z^{-n_o}	$\forall z$
$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u(-n-1)$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
$\cos(\Omega_0 n)u(n)$	$\frac{1-z^{-1}\cos\Omega_0}{z^{-2}-2z^{-1}\cos\Omega_0+1}$	$\|z\| > \|1\|$
$\sin(\Omega_0 n)u(n)$	$\frac{z^{-1}\sin\Omega_0}{z^{-2}-2z^{-1}\cos\Omega_0+1}$	$\|z\| > \|1\|$
$a^n\cos(\Omega_0 n)u(n)$	$\frac{1-z^{-1}a\cos\Omega_0}{a^2z^{-2}-2az^{-1}\cos\Omega_0+1}$	$\|z\| > \|a\|$
$a^n\sin(\Omega_0 n)u(n)$	$\frac{z^{-1}a\sin\Omega_0}{a^2z^{-2}-2z^{-1}a\cos\Omega_0+1}$	$\|z\| > \|a\|$