JUNE 23, 2020

Lastname	Name	D.N.I.

Length: 3 hours

- Justify and explain your answers.
- The only authorized resources are the official tables and blue or black pens. Books, calculators, pencils, etc are *not* allowed.
- Write the response to each exercise on a different piece of paper

Exercise 1 (3,5 points)

Let us consider the the following RLC circuit , which is described by the following differential equation:

$$i''(t) + \frac{R}{L}i'(t) + \frac{1}{LC}i(t) = \frac{1}{L}v'(t)$$

and assuming zero initial conditions. The signal $v(t)$ is set by a source and it can be considered as the input to the system.

1. Determine the transfer function by means of the Laplace transform. **(0.25p)**
2. Let $R = 3$, $L = 1$ and $C = \frac{1}{2}$, determine the impulse responses compatible with the differential equation. Which one models the system under analysis? **(0.75p)**
3. Let $\alpha = \frac{R}{2L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$. Study the stability of the system for the cases $\alpha < \omega_0$ and $\alpha = \omega_0$. **(1p)**
4. Let $R = 4$, $L = 1$ and $C = \frac{1}{4}$, if the input corresponds to a sudden voltage $v(t) = u(t)$, find the output of the system. **(0.75p)**
5. Let us consider a sinusoidal steady-state where the input is $v(t) = \cos(\omega_1 t)$ with $\omega_1 \neq \omega_0$, determine the output if $\alpha = \omega_0$. **(0.75p)**

Exercise 2 (2 points)

Calculate the following operations and simplify it as much as possible:

1. Determine the following convolution: $c(x) = f(x) * g(x)$ with $f(x) = \sin(\pi x)\Pi\left(\frac{x-1}{2}\right)$ and $g(x) = \Pi\left(\frac{x-2}{2}\right)$ **(0.5p)**
2. Determine the convolution: $c(x) = \left(\frac{d^2 f(x)}{dx^2}\right) * g(x)$ with $f(x) = \Pi(\frac{2x-1}{2})$ y $g(x) = e^{3x}$ **(0.5p)**
3. Determine the energy of the signal $c(n) = x(n) * y(n)$ with $x(n) = \Pi_5(n-2)$ and $y(n) = \Pi_5(n-4) + \Pi_5(n-11)$ **(0.5p)**
4. Determine the energy of the signal $c(x) = f(x) * g(x)$ with $f(x) = 6\text{sinc}(3x)$ and $g(x) = \frac{2}{\pi}\text{sinc}^2(2x)$ **(0.5p)**

Exercise 3 (1,5 points)

The convolution of discrete-variable signals meets the commutative and distributive properties. Nevertheless, the associative property requires some special conditions which are not always met.

1. Show that the associative property is met for well-behave signals: **(0.5p)**

$$x(n) * (y(n) * z(n)) = (x(n) * y(n)) * z(n)$$

2. Let $x(n) = u(n)$, $y(n) = \delta(n) - \delta(n-1)$ y $z(n) = A$, determine $c_1(n) = [x(n) * y(n)] * z(n)$ and $c_2(n) = x(n) * [y(n) * z(n)]$. Is the associative property met in this case? **(0.5p)**

3. Let $x(n) = \delta(n) - a\delta(n-1)$, $y(n) = a^n u(n)$, $|a| < 1$, y $z(n) = e^{j8n}$, determine $c_1(n) = [x(n) * y(n)] * z(n)$ and $c_2(n) = x(n) * [y(n) * z(n)]$. Is the associative property met in this case? **(0.5p)**

Some trigonometric identities of (potential) interest

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$
$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$

$$\sin(x)\sin(y) = \frac{\cos(x-y)-\cos(x+y)}{2}$$
$$\cos(x)\cos(y) = \frac{\cos(x+y)+\cos(x-y)}{2}$$
$$\sin(x)\cos(y) = \frac{\sin(x+y)+\sin(x-y)}{2}$$