Analysis of the Young's experiment by means of the Fourier Transform

1 Goals

This session aims at analyzing classical experiment, which was usually to illustrate the wave theory of light and, subsequently, the wave-particle duality of light. For this purpose, we will resort to the tools, which have been studied in this course. First, the theoretical background will be detailed by resorting to modern tools in contrast to the ones used in the original experiment, showing that this experiment can be easily analyzed. Next, an experimental demonstration will be performed.

The specific goals of this session are:

1. To be able to identify signals transforms when they rise in real-world problems.
2. To become familiar with transforms along the spatial domain.
3. To be able to estimate physical parameters from spectral information.

2 Theoretical background

In this session, the analysis of a beam light partly blocked and the resulting scattered light will be considered. For this purpose, the Fraunhofer diffraction equation, which models the previous problem, will be introduced. Next, the experiment performed by Thomas Young in 1801, which entails blocking the light with two narrow apertures (slits), and the corresponding theoretical model, will be considered.

2.1 Description of the system

Fig. 1 shows the problem under consideration, in which the a monochromatic light (i.e., composed of a single wavelength λ) is partly blocked by screen with several slits. The amplitude of the light along the aperture plane is denoted by $a(x')$. After the blocking screen, the light propagates so that the amplitude at a given distance L is denoted by $I(x)$.

The relation between the input $a(x')$ and output $I(x)$ of the system is given Fraunhofer diffraction equation:

$$I(x) \propto \int_{-\infty}^{\infty} a(x') e^{-jk\frac{xx'}{L}} dx'$$

where $k = \frac{2\pi}{\lambda}$ is the wavenumber at the working wavelength λ.

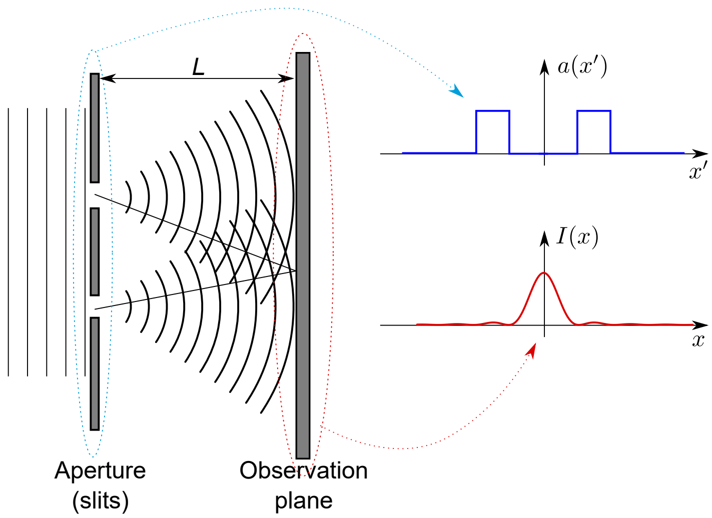


Figure 1: Description of the light diffraction.

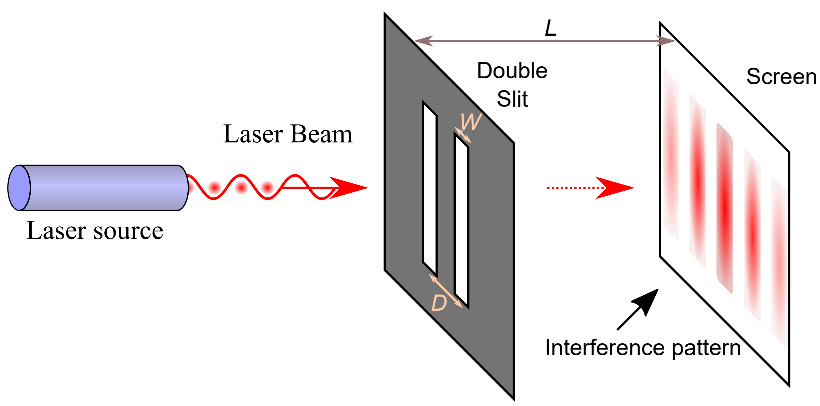


Figure 2: Double-slit experiment. The original experiment was similar but sunlight filtered through a pinhole was used instead of a laser beam.

Although an in-depth study is out of the scope of this course, it is important to remark that both signals are complex-valued signals as they described phasors. Moreover, it is also useful to note that our eyes are sensitive not to the complex amplitude $I(x)$ but to the intensity $|I(x)|^2$.

TASK

1. Express the previous equation in terms of known transforms.

2.2 Young's experiments

The Young's experiment, also known as double-slit experiment, is a classical demonstration of the *wave-particle duality*. The experiment was first performed by Thomas Young in 1801. A possible implementation using modern components is illustrated in Fig. 2.

In the basic version of the experiment, a coherent light sources illuminates a plate pierced by two parallel slits generating an interference pattern (also known as fringe pattern) at a given observation plane. As previously described, this interaction can be modeled by means of the Fraunhofer diffraction equation.

Given a wavelength $\lambda = 532\,\text{nm}$, which corresponds to the green color, and an observation plane at $L = 3.9\,\text{m}$, perform the following tasks:

TASK

1. Determine the light intensity $|I(x)|^2$ if the experiment involves a single slit with a width of $W = 100\,\mu\text{m}$.
2. Determine the light intensity $|I(x)|^2$ in case of considering two slits of width $W = 100\,\mu\text{m}$ separated by a distance $D = 1\,\text{mm}$.

3 Experimental demonstration

Next, an experimental demonstration will be performed. From the measurements, you will be able to obtain some of the parameters from the previous equations. Besides, Fig. 3 shows some reference results, which can give you an idea of the expected measurements.

You can reproduce the experiment at home by using aluminum foil, a cutter knife, a ruler and a cheap laser. With the help of the ruler, carry out two cuts to create two parallel slots as close as possible (1 mm or so) in the aluminum foil. The width of the slot will be roughly the width of the cutter knife. The recommended length of the lines is at least 1 cm. After that, illuminate the slots with the laser and see the projection at a distance of 4-5m.

You can also follow other tutorials available on the internet:
https://www.youtube.com/watch?v=kKdaRJ3vAmA

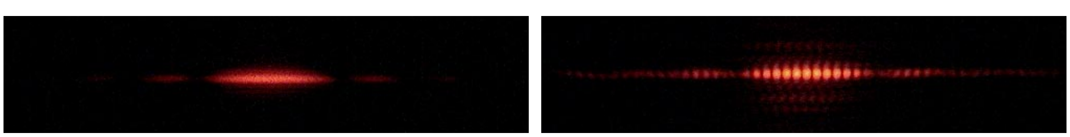
Once you have replicated the experiment, you will have to:

TASK

1. Measure the distance L.
2. Given the wavelength (green: $\lambda = 532\,\text{nm}$, red: $\lambda = 650\,\text{nm}$), determine the slit separation D from the signal $|I(x)|^2$.
3. Determine the width of the slits W from the signal $|I(x)|^2$.

OPTIONAL

1. Repeat the last two tasks for a laser working at a different wavelength.



(a) (b)

Figure 3: Experimental results for (a) single-slit and (b) double-slit experiments.