Problem-solving session

Discrete variable: LSI systems and FS

Jaime Laviada

Área de Teoría de la Señal y Comunicaciones
Universidad de Oviedo

Exercise #1

Suppose we are given the following facts about the signal $x_0(n)$:

1. $x_0(n)$ is periodic, with period $N_0 = 6$
2. $\sum_{n=0}^{5} x_0(n) = 2$
3. $\sum_{n=2}^{7} (-1)^n x_0(n) = 1$
4. $x_0(n)$ has the minimum average power among the set of signals satisfying the preceding three conditions.

Determine the signal $x_0(n)$.

Exercise #2

(based on exercise 4.12 from Vol. ST-II by Emilio Gago)

a) In the case of the continuous-variable realm, the following relations between the distribution $\delta(x)$ and the unit-step function $u(x)$ are well-known:

$$\int_{-\infty}^{x} \delta(y)dy = u(x) \qquad u'(x) = \delta(x).$$

Determine the equivalent relations in the case of discrete-variable functions.

b) Let $h(n)$, be the impulse response of LSI system such that $h(n) = u(n)$. Determine the impulse response of the inverse system.
Tip: exploit the equations from the previous question.

Exercise #2

c) Consider two identical systems, which are cascaded, and let $h(n) = u(n)$ be the impulse response of each subsystem:

i) Determine the impulse response of the entire system

ii) Check that the previous system can also be modeled by means of the operator $\mathbf{F}(\cdot) = \sum_{k=-\infty}^{n} \sum_{m=-\infty}^{k} (\cdot)$

iii) Find the impulse response of the inverse system.

iv) Let $x_0(n) = \cos(\Omega_0 n)$ with $\frac{\Omega_0}{2\pi}$ a irrational number, be the input to the system. The output to that input is known to be $y_0(n) = A\cos(\Omega_0 n + \phi)$ with $A = \frac{1}{2(1-\cos\Omega_0)}$ and $\phi = -2\tan^{-1}\left(\frac{\sin\Omega_0}{1-\cos\Omega_0}\right)$.
Validate that the inverse system is able to retrieve the original input $x_0(n)$ from $y_0(n)$ by working in the real domain.