Problem solving session

Impulse response of linear and shift-invariant systems

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Exercise 1: Stability of the LSI systems

(Based on exercise 2.49 from Signals and Systems by A. Oppenheim)
In the lectures, we have described how the stability of a LSI system can be assessed in based on its impulse response. In particular, absolute integrability is a *necessary and sufficient condition* for stability:

$$\text{LSI system stable} \Leftrightarrow \int_{-\infty}^{\infty} |h(x)|\, dx < \infty$$

a) Show that the next requirement is a sufficient condition:

$$\int_{-\infty}^{\infty} |h(x)|\, dx < \infty \Rightarrow \text{LSI system stable}$$

Exercise 1: Stability of the LSI systems

In order to show that it is also a necessary condition, we will tailor a bounded input signal which yields an unbounded output signal.
Thus, let us consider a LSI system whose impulse response is not absolutely integrable, i.e.,

$$\int_{-\infty}^{\infty} |h(x)|\, dx \to \infty$$

moreover, let us consider the following input:

$$f(x) = \begin{cases} 0 & \text{si } h(-x) = 0 \\ \frac{h^*(-x)}{|h(-x)|} & \text{si } h(-x) \neq 0 \end{cases}$$

b) Does this input signal represent a bounded input? If so, what is the smallest number A such that:

$$|f(x)| \leq A \quad \forall x?$$

c) Calculate the output at $x = 0$ for this particular choice of input. Does the result prove that the contention that absolute integrability is a necessary condition for stability?

Exercise 2: Properties of the impulse response

In this exercise, we will consider again the problem with the following relationship between input/output:

$$g(x) = \int_x^{\infty} f(y) e^{x-ay} dy \text{ with } a \geq 1$$

however, it will be analyzed in terms of the impulse response or the collection of impulse responses.

a) Calculate the impulse response or the collection of impulse responses. Formulate the result in terms of $u(x)$ and validate reasonably the obtained result.

b) Calculate the response to the unit step $u(x)$ using two different methods. Compare the results and represent the results graphically for some values of $a > 1$.

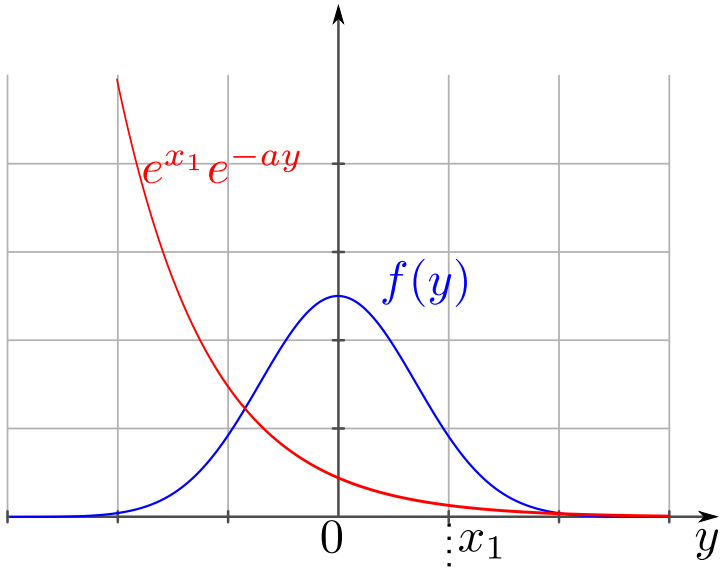
Exercise 2: Properties of the impulse response

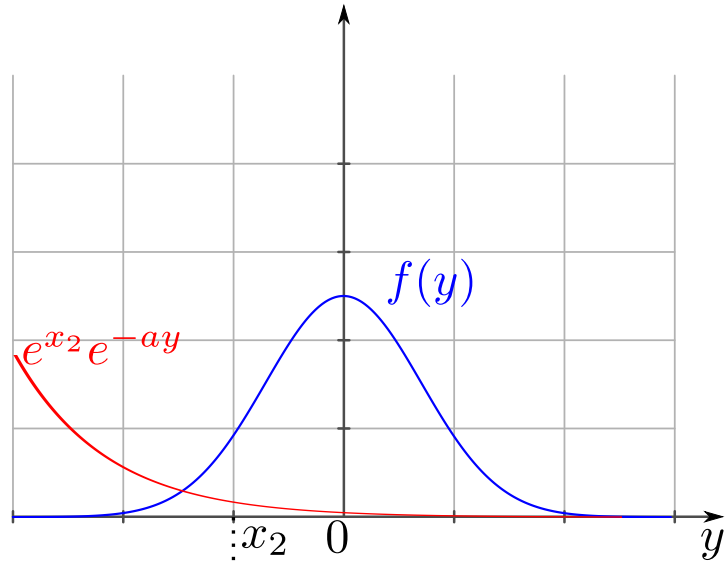
c) Explain what happens if $a = 1$. Compare the results for the previous a) and b) sections of this exercise.

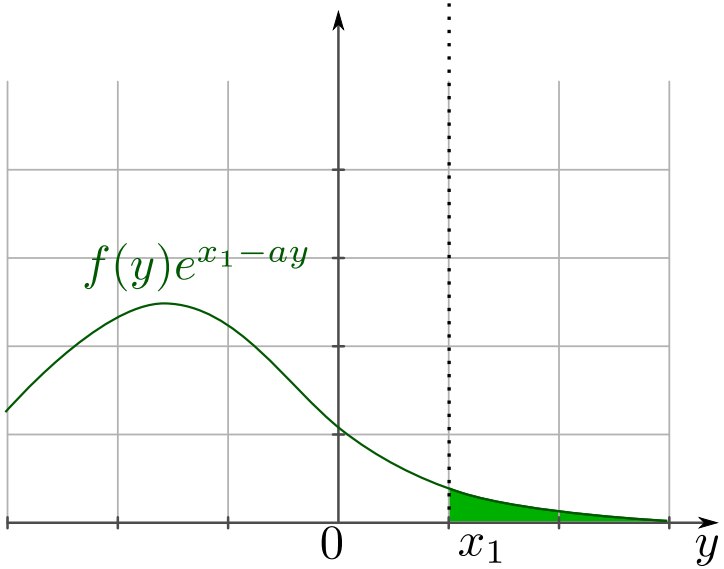
d) Let $a = 1$, by using the previous results, evaluate the following properties of the system: memory, causality and stability.

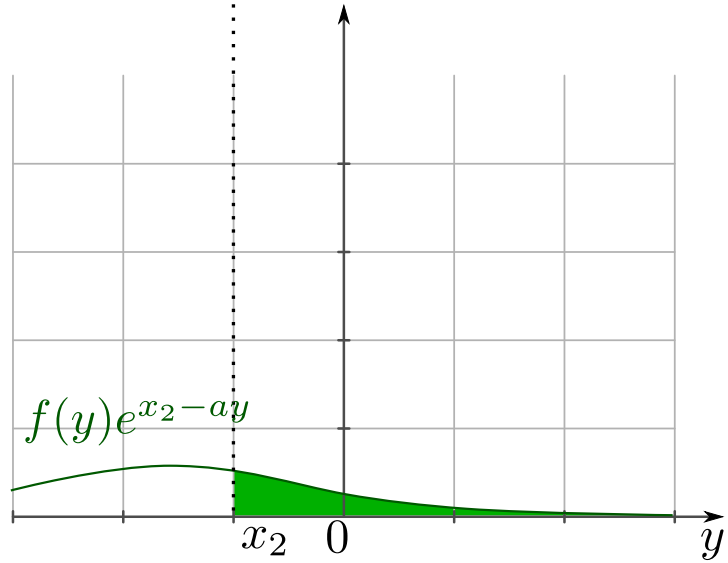
e) Assess the invertibility of the system. For this purpose, it is advised to firstly evaluate $h(x) * \delta'(x)$.

Solution









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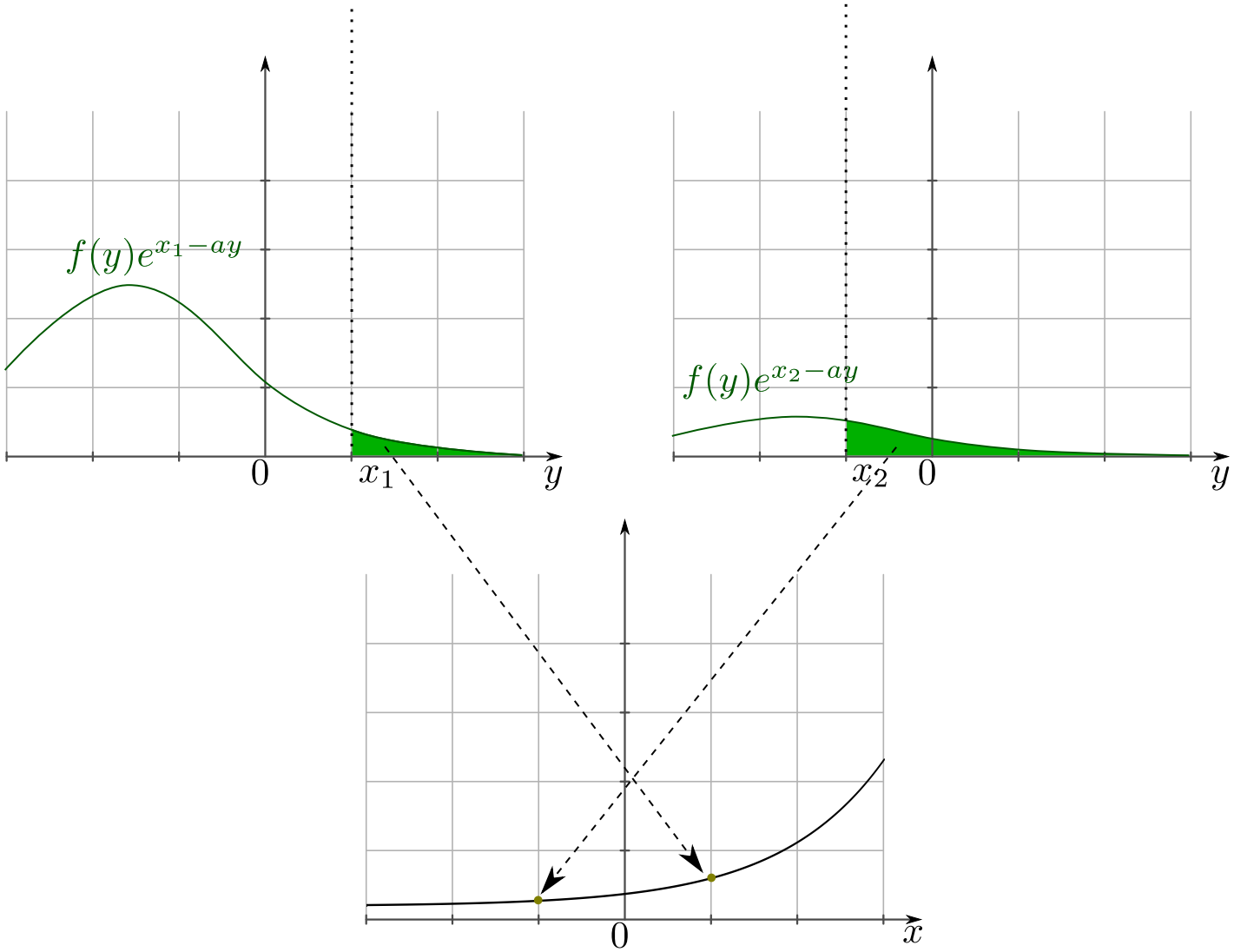
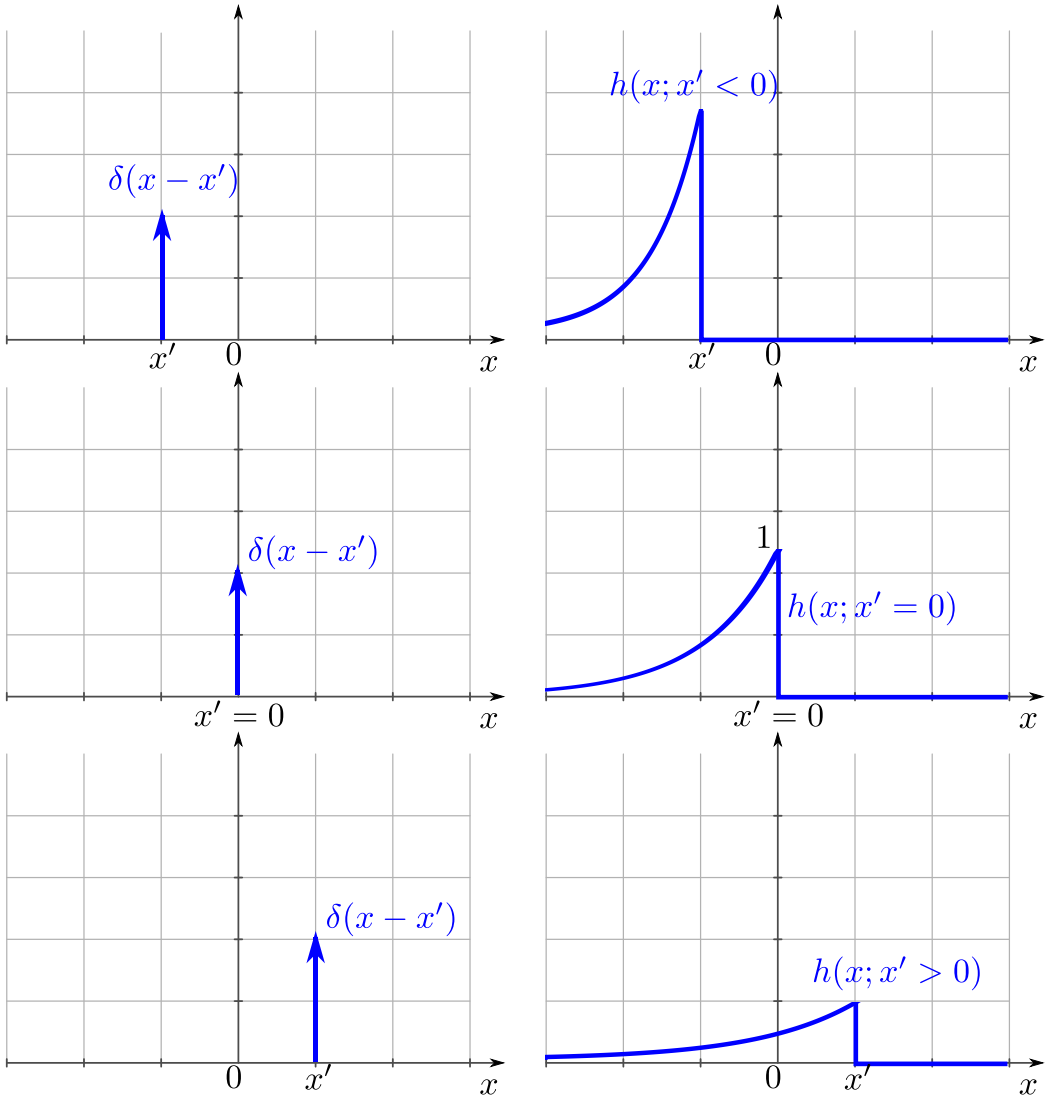


Solution


$\delta(x - x')$
x'
0
x
$h(x; x' < 0)$
x'
0
x
$\delta(x - x')$
$x' = 0$
x
1
$h(x; x' = 0)$
$x' = 0$
x
$\delta(x - x')$
0
x'
x
$h(x; x' > 0)$
0
x'
x

Solution