Problem-Solving Session

Convolutions

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Exercise 1: Properties of the distributions

Two distributions $D_1(x)$ and $D_2(x)$ are equal if

$$\int_{-\infty}^{\infty} g(x)D_1(x)dx = \int_{-\infty}^{\infty} g(x)D_2(x)dx, \quad \forall g(x).$$

According to the previous definition, prove the next properties:

$f(x)\delta(x-x_0) = f(x_0)\delta(x-x_0)$	$f(x)\delta'(x-x_0) =$ $f(x_0)\delta'(x-x_0) - f'(x_0)\delta(x-x_0)$
$x\delta(x) = 0$	$x\delta'(x) = -\delta(x)$
$f(x) * \delta(x-x_0) = f(x-x_0)$	$f(x) * \delta'(x-x_0) = f'(x-x_0)$
$\delta(ax-x_0) = \frac{1}{\lvert a\rvert}\delta(x-\frac{x_0}{a})$	$\delta'(ax-x_0) = \frac{1}{\lvert a\rvert}\frac{1}{a}\delta'(x-\frac{x_0}{a})$
$\delta(x) = \delta(-x)$	$\delta'(x) = -\delta'(-x)$

Exercise 2: Sequence of functions

(Based on exercise 4.6 from the book by Emilio Gago Vol. ST-II)
Analysis of $\delta(x)$ and $u(x)$:

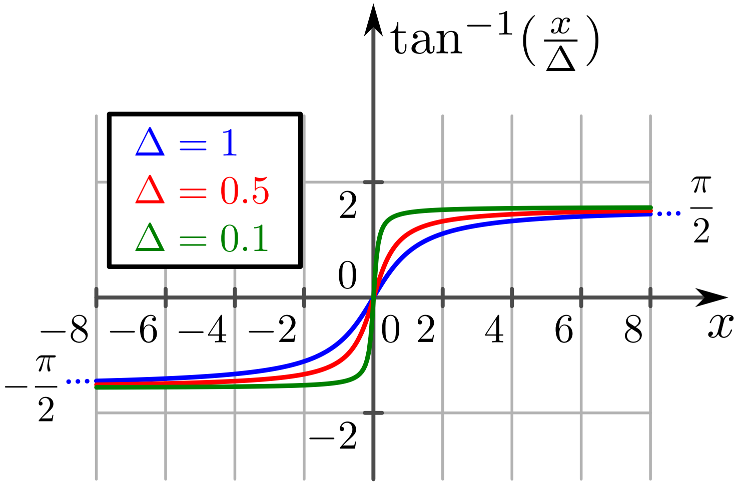
a) Taking into account the shape of $\tan x$, find a function $p_\Delta(x)$ such that $d(u(x), \lim_{\Delta\to 0} p_\Delta(x)) = 0$.

b) Tailor a sequence of functions $p_n(x)$ such that $d(u(x), \lim_{n\to\infty} p_n(x)) = 0$.

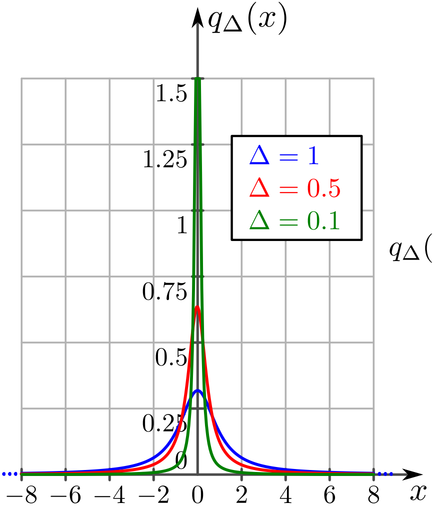
c) Taking into account the relation between $\delta(x)$ and $u(x)$ find a new function $q_\Delta(x)$ from $p_\Delta(x)$ such that it enable us to model the distribution $\delta(x)$ when it approaches zero, i.e., $\Delta \to 0$.

d) Find a sequence of functions defining the first derivative of $\delta(x)$, i.e., $\delta'(x)$

Exercise 2: Sequence of functions



Exercise 2: Sequence of functions



$$q_\Delta(x) = \frac{1}{\pi} \frac{\Delta}{x^2 + \Delta^2}$$