Problem-Solving Session

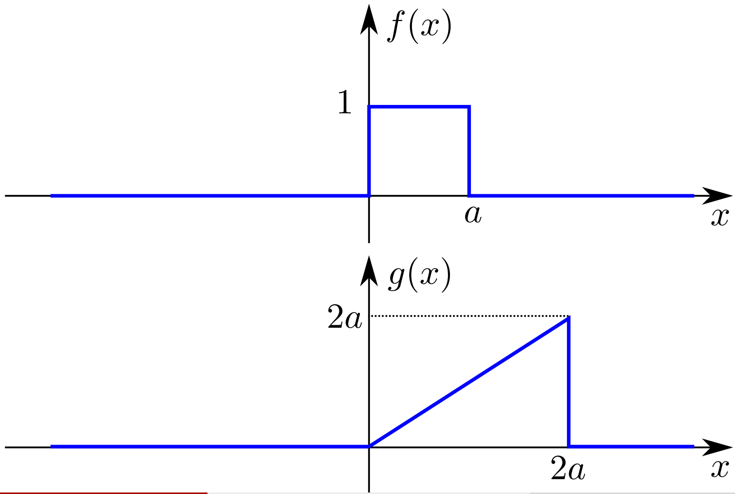
Convolutions

Jaime Laviada

Área de Teoría de la Señal y Comunicaciones
Universidad de Oviedo

Exercise 1: Computing a convolution from its graphic representation

Compute the convolution of the two following signals:



Exercise 2: Computing a convolution from its analytical representation

Compute the convolution of $f(x) = x\Pi(x - \frac{1}{2})$ and $g_0(x) = \cos(2\pi x)$. It is recommended to sketch the signals.

Exercise 3: Associative property of the convolution

(Based on exercise 2.43 from Signals and Systems by Oppenheim)
One of the important properties of convolution, in both the discrete- and continuous-variable, is the associative property. In this problem, we will check and illustrate this property.

a) Prove the equality

$$[f(x) * h(x)] * g(x) = f(x) * [h(x) * g(x)]$$

by showing that both sides equal:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x')h(y')g(x-x'-y')dx'dy'$$

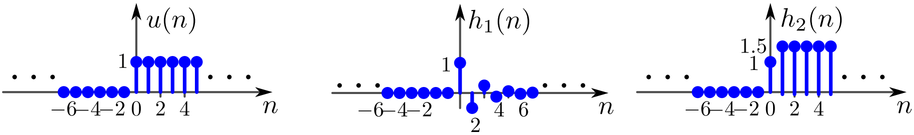
Exercise4: Associative property of the convolution

Consider three discrete-variable signals $h_1(n)$, $h_2(n)$ and $x(n) = u(n)$ defined as in the figure below

$$h_1(n) = \left(-\frac{1}{2}\right)^n u(n) \text{ and } h_2(n) = u(n) + \frac{1}{2}u(n-1)$$

i) Compute $y(n)$ as $y(n) = [x(n) * h_1(n)] * h_2(n)$

ii) Compute $y(n)$ as $y(n) = x(n) * [h_1(n) * h_2(n)]$



Exercise 4: Associative property of the convolution

iii) Consider the following signals

$$h_1(n) = \sin 8n$$

$$h_2(n) = a^n u(n), |a| < 1,$$

$$x(n) = \delta(n) - a\delta(n-1).$$

Compute $y(n) = x(n) * h_1(n) * h_2(n)$. (Tip: use the associative and commutative properties to keep the simple as simple as possible).