Problem-Solving Session

Properties of the systems

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Exercise 1: Inverse systems

(based on exercise 4.20 from book by Emilio Gago)
Let $\mathbf{F}$ and $\mathbf{F}^{-1}$ two continuous-variable operators modeling two different systems. Show that if $\mathbf{F}$ is a linear operator, then the inverse system $\mathbf{F}^{-1}$ is also linear. For this purpose, it is recommended to resort to the definition of linear system.

Exercise 2: Linear system with integral operator

(Based on exercise 4.18 from book by Emilio Gago)
Let a linear system, which relates input $f(x)$ and output $g(x)$ by means of the following integral equation

$$g(x) = \int_x^{\infty} f(y) e^{x-ay} dy, \text{ with } a \geq 1$$

With respect to the previous system:

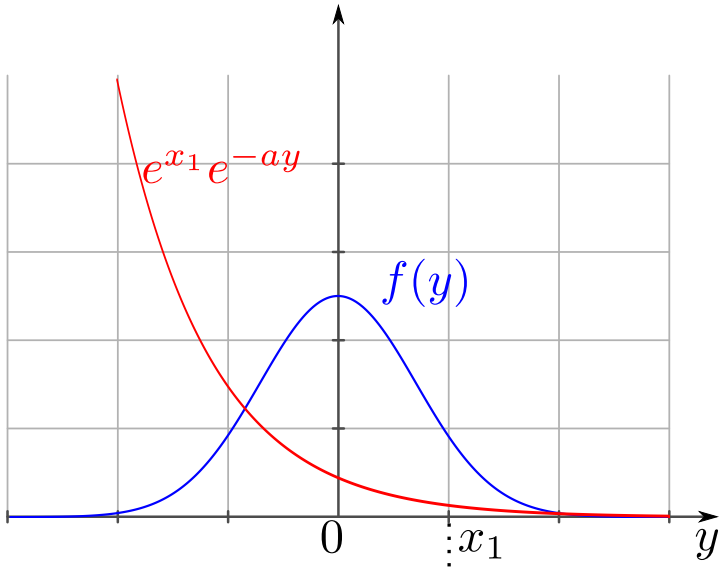
a) Study graphically the previous system and characterize it for at least two particular output values, i.e., $g(x_1)$ y $g(x_2)$.

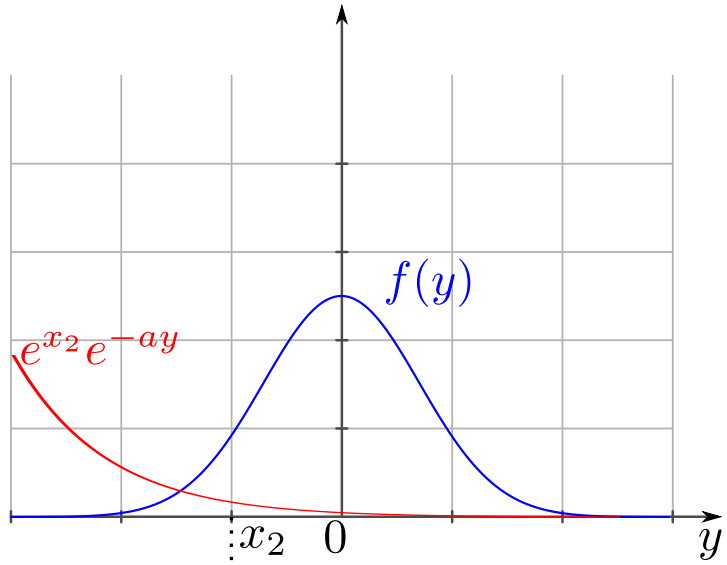
b) Analyze the shift-invariance property.

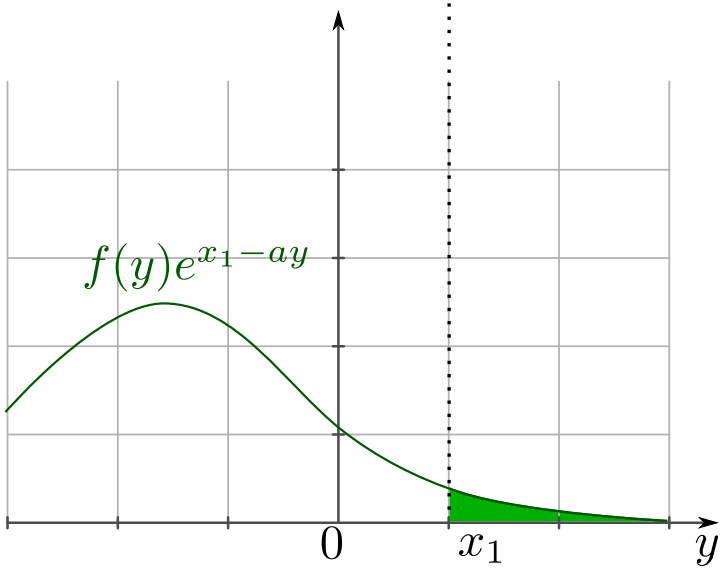
c) Let $a = 1$ and, resorting to the previous results, identify the memory, causality and stability properties.

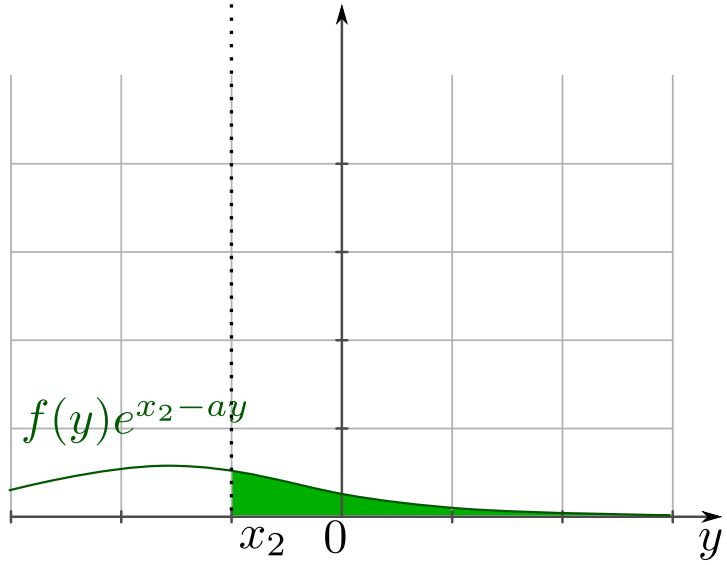
d) Compute the inverse operator in case of $a \geq 1$. For this purpose, examine the result of the following operation on the output: $\frac{dg(x)}{dx}$.

Solution









Solution