Problem-Solving Session

Distance definitions in Space Signals

Jaime Laviada

Área de Teoría de la Señal y Comunicaciones
Universidad de Oviedo

Exercise 1: Distance definition

(Based on exercise 4.1 from the book by Emilio Gago Vol. ST-II)

a) Perform a graphical analysis and/or analytical analysis of the following (bivariate) applications:

$$d_1 : \mathcal{V}_{\mathcal{K}} \times \mathcal{V}_{\mathcal{K}} \to \mathbb{R}$$
$$(f(x), g(x)) \mapsto d_1(f(x), g(x)) = \int_{-\infty}^{\infty} |f(x) - g(x)|\, dx$$

$$d_2 : \mathcal{V}_{\mathcal{K}} \times \mathcal{V}_{\mathcal{K}} \to \mathbb{R}$$
$$(f(x), g(x)) \mapsto d_2(f(x), g(x)) = \left[\int_{-\infty}^{\infty} |f(x) - g(x)|^2\, dx\right]^{1/2}$$

$$d_3 : \mathcal{V}_{\mathcal{K}} \times \mathcal{V}_{\mathcal{K}} \to \mathbb{R}$$
$$(f(x), g(x)) \mapsto d_3(f(x), g(x)) = \sup\left[|f(x) - g(x)|\right]$$

b) Determine which ones are valid definitions of (pseudo)distance definitions as well as which ones satisfy that $d(f(x), g(x)) = 0 \Leftrightarrow f(x) = g(x)$.

c) Determine what kind of functions have a distance zero for each case.

Exercise 1: Norm definitions

d) Determine if the following definitions are properly defined norms:

$$\|\cdot\|_1 : \mathcal{V}_{\mathcal{K}} \to \mathbb{R}$$
$$f(x) \mapsto \|f(x)\|_1 = \int_{-\infty}^{\infty} |f(x)|\, dx$$
$$\|\cdot\|_2 : \mathcal{V}_{\mathcal{K}} \to \mathbb{R}$$
$$f(x) \mapsto \|f(x)\|_2 = \left[\int_{-\infty}^{\infty} |f(x)|^2\, dx\right]^{1/2}$$
$$\|\cdot\|_\infty : \mathcal{V}_{\mathcal{K}} \to \mathbb{R}$$
$$f(x) \mapsto \|f(x)\|_\infty = \sup\left[|f(x)|\right]$$

Note that the Minkowski inequality establishes that:

$$\left[\int_{-\infty}^{\infty} |f(x)+g(x)|^p\, dx\right]^{1/p} \leq \left[\int_{-\infty}^{\infty} |f(x)|^p\, dx\right]^{1/p} + \left[\int_{-\infty}^{\infty} |g(x)|^p\, dx\right]^{1/p}$$