Problem-solving session

Plotting complex functions

Jaime Laviada

Área de Teoría de la Señal y Comunicaciones
Universidad de Oviedo

Exercise #1: Plotting complex functions

(based on exercise 4.5 from Vol. ST-II by Emilio Gago)

a) Study the following functions from analytical and graphical points of view:

$$e(t;m) = e^{jm\omega_0 t},\ \ \omega_0 = \frac{2\pi}{T_0} \text{ and } m \in \mathbb{Z}.$$

b) Use the previous functions to perform the following operations as well as to graphically analyze the result:

i. $u_0(t) = e(t;0) + e(t;1)$
ii. $v_0(t) = e(t;2) + e(t;-1)$
iii. $w_0(t) = e(t;3) + e(t;-3) + e(t;0)$
iv. $y_0(t) = e(t;1) + e(t;2) + e(t;3)$

Exercise #1: Plotting complex functions

c) Analyze and plot the following functions:

i. $f_0(t) = \sin(\omega_0 t + \varphi_0)$

ii. $g_0(t) = \sin(m\omega_0 t + \varphi_0)$

iii. $h_0(t) = \sin(m\omega_0 t + \varphi(t))$, con $\varphi(t) = \omega_n t$, $\omega_n \in \mathbb{R}$ y $\omega_0 > 0$. Study the possible values of the new period of the signal as a function of ω_n. Tip: plot the new value of the period T_{0n} in terms of ω_n.

iv. Let $u(t)$ be a well-behaved function over the interval $(-\infty, \infty)$ whose phase is constant $\varphi_u(t) = \varphi_0 = cte.$, study the following function from analytical and graphical points of view

$$v(t) = u(t)e(t;1)$$

Exercise #1: Plotting complex functions

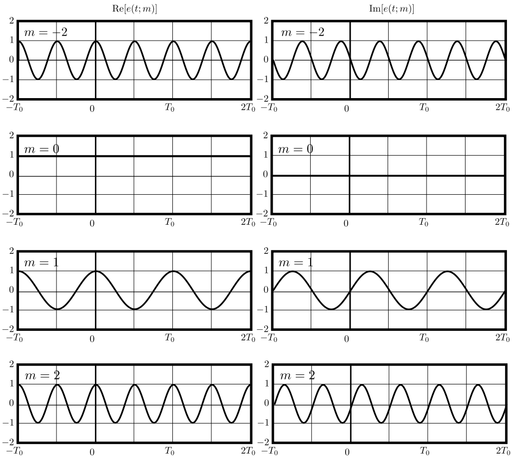

Figure: Real and imaginary parts of the harmonic functions $e(t;m) = \exp(jm\omega_0 t)$ for $m = -2, 0, 1$ and 2.

Exercise #1: Plotting complex functions

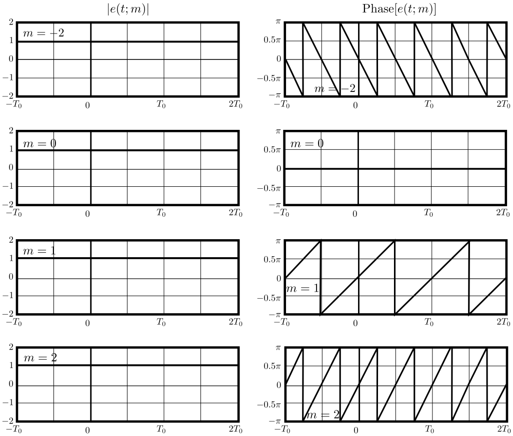

Figure: Amplitude and phase of the harmonic functions $e(t;m) = \exp(jm\omega_0 t)$ for $m = -2, 0, 1$ and 2.

Exercise #1: Plotting complex functions

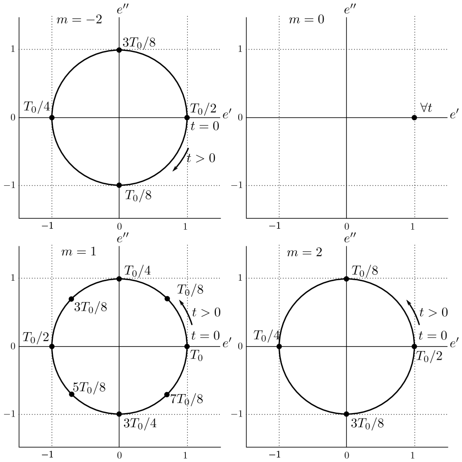

Figure: Representation in the complex plane of (e', e'') over a fundamental period of the harmonic function $e(t;m) = \exp(jm\omega_0 t)$ for $m = -2, 0, 1$ y 2.

Exercise #1: Plotting complex functions

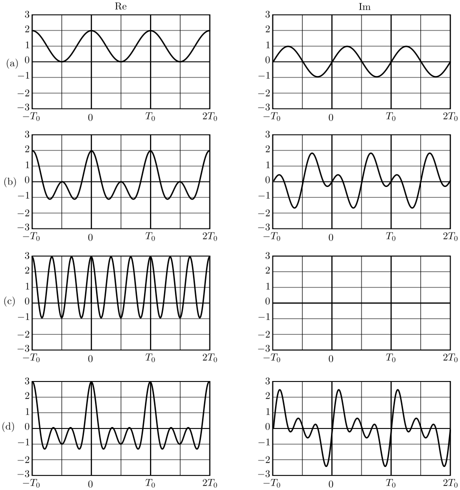

Figure: Representation of the signals (a) $u_0(t)$, (b) $v_0(t)$, (c) $w_0(t)$, (d) $y_0(t)$, in terms of the real and imaginary parts.

Exercise #1: Plotting complex functions

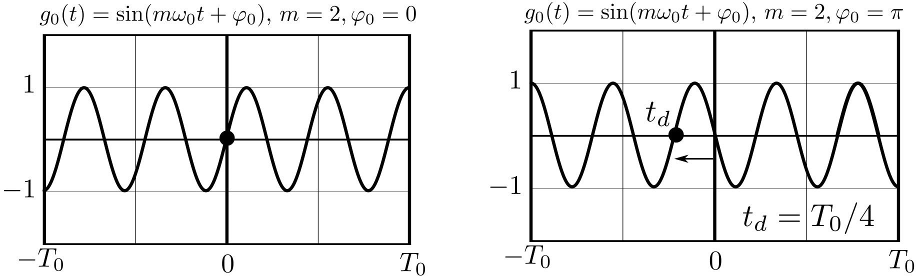

Figure: Representation of a sine function with frequency $2f_0$ and phase (a) $\varphi_0 = 0$ y (b) $\varphi_0 = \pi$.

Exercise #1: Plotting complex functions

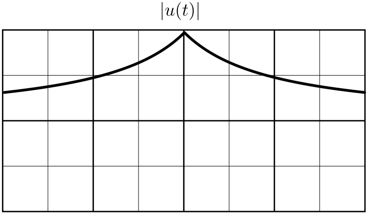

Figure: Representation of (a) $u(t)$, (b) $\mathrm{Re}\{u(t)\exp(j\omega_0 t)\}$ and (c) $\mathrm{Im}\{u(t)\exp(j\omega_0 t)\}$.

Exercise #1: Plotting complex functions

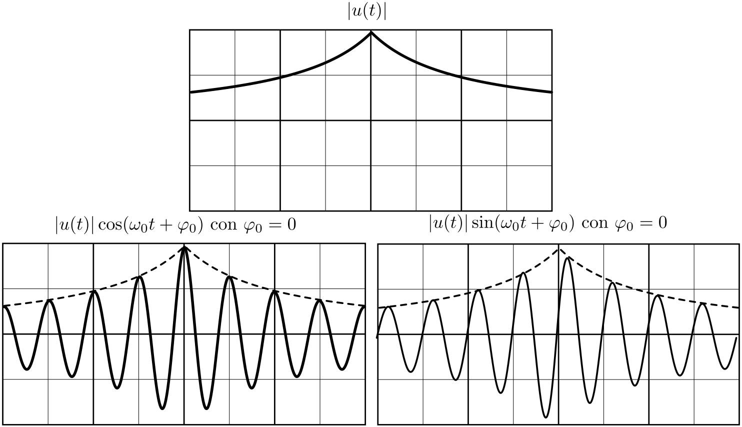

Figure: Representation of (a) $u(t)$, (b) $\mathrm{Re}\{u(t)\exp(j\omega_0 t)\}$ and (c) $\mathrm{Im}\{u(t)\exp(j\omega_0 t)\}$.