Relevant Discrete-Variable Signals

Tema 8

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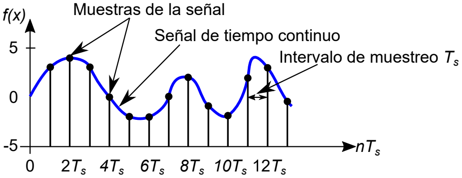
Outline

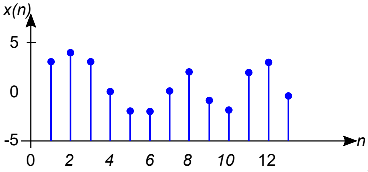
Discrete-Variable Signals

- The independent variable is an integer number

$$x(n),\ n \in \mathbb{Z}$$

- They are often referred to as sequences.
- They are used to model discrete processes (e.g., daily stock marker values) or due to sampling a continuous-variable signal.
- The latter is particularly relevant because it is the tool used by the computers to numerically process continuous-variable signals.





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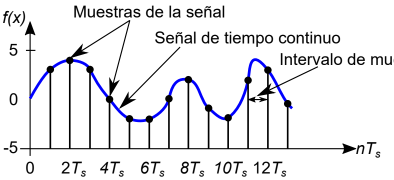

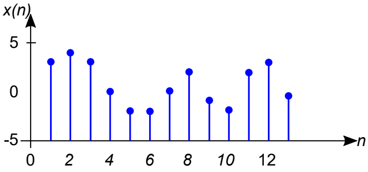

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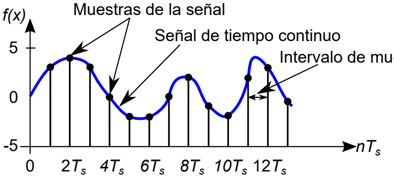

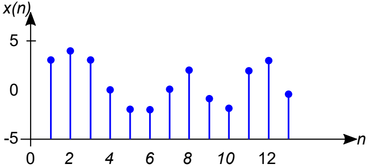

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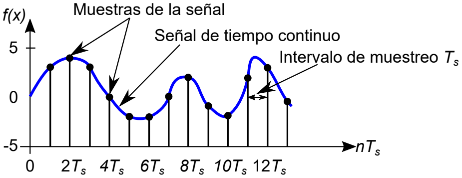
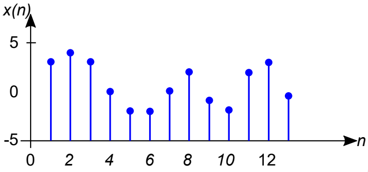




Signal spaces for discrete-variable signals

- In a similar way to the continuous-variable case, two different signal spaces will be considered.

- $D(\mathbb{Z})$ is the space signals of aperiodic signals with $n \in (-\infty, \infty)$.
- The inner product in $D(\mathbb{Z})$ is usually defined by:

$$\langle x(n), y(n) \rangle = \sum_{n=-\infty}^{\infty} x(n) y^*(n)$$

- $D(N_0)$ is the space signal of periodic signals, whose period is N_0
- The inner product in $D(N_0)$ is usually defined by:

$$\langle x_0(n), y_0(n) \rangle = \sum_{n=\langle N_0 \rangle} x_0(n) y_0^*(n)$$

- In both cases, norms and distances can be defined as usually.
  - If the norm induced by the inner product, and the distance induced by the norm are considered, then we have:

$$\|x(n)\|^2 = \langle x(n), x(n) \rangle \quad d\left(x(n), y(n)\right) = \|x(n) - y(n)\|$$

Energy and Power

Quantity	Finite-length	Infinite-length
Instantaneous Power	$\mathbf{P}[x(n)] = \lvert x(n)\rvert^2 = x(n)x^*(n)$	
Energy	$E[x(n)] = \sum_{n=N_1}^{N_2} \mathbf{P}[x(n)]$	$E[x(n)] = \sum_{n=-\infty}^{\infty} \mathbf{P}[x(n)]$
Average Power	$\langle \mathbf{P}[x(n)] \rangle = \frac{1}{N} \sum_{n=N_1}^{N_2} \mathbf{P}[x(n)]$ $N = N_2 - N_1 + 1$	$\langle \mathbf{P}[x(n)] \rangle = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=N}^{N} \mathbf{P}[x(n)]$

- These definitions are also valid for periodic signals $x_0(x)$. However, it is usually more convenient to express the average power as: $\langle \mathbf{P}[x_0(n)] \rangle = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} \mathbf{P}[x_0(n)]$
- In addition, the energy of periodic signals is infinite and, therefore, it is better to use the energy per period: $E_0[x_0(n)] = \sum_{n=\langle N_0 \rangle} \mathbf{P}[x_0(n)] = N_0 \langle \mathbf{P}[x_0(n)] \rangle$

Discrete-Variable Convolution

- In a similar way to the CV case, two kind of convolutions are defined.
- Aperiodic convolution (or, simply, convolution):

$$\boxed{c(n) = x(n) * y(n) = \sum_{m=-\infty}^{\infty} x(m)y(n-m)}$$

- Periodic convolution, for signals in $D(N_0)$:

$$\boxed{c_0(n) = x_0(n) * y_0(n) = \sum_{m=\langle N_0\rangle} x_0(m)y_0(n-m)}$$

- Note that the result $c_0(n)$ is also in $D(N_0)$.
- Both convolutions meet again the following properties: commutativity, associativity and distributivity over addition:

$$\boxed{x(n) * y(n) = y(n) * x(n)} \quad \boxed{x(n) * (y(n) * z(n)) = (x(n) * y(n)) * z(n)}$$

$$\boxed{x(n) * (y(n) + z(n)) = x(n) * y(n) + x(n) * z(n)}$$

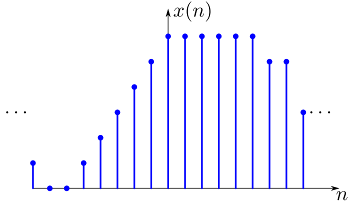
Usual operations with discrete-variable signals

- Operations with discrete-variable signals are very similar to those with continuous-variable signals except for a few (but relevant) details.
- Addition and product of signals are defined as a pointwise operations again.
- In a similar fashion, scalar multiplication is also performed pointwisely.
- The reversal and shift operations are also analogous to the CV case.
- Nevertheless, two new operations related to the scaling of the independent variable are defined: decimation and zero padding.
    - In both cases, the scaling factor must be an integer number $N \in \mathbb{Z}$

Transformations of the independent variable - I

Shift

$x(n) \rightarrow x(n - n_0)$



Reversal

$x(n) \rightarrow x(-n)$

Independent variable scaling: Decimation

$x(n) \rightarrow x(pn),\ p \in \mathbb{Z}$

Independent variable scaling: Zero-padding

$$x(n) \rightarrow x(n/p) = \begin{cases} x(m) & n = mp \\ 0 & n \neq mp \end{cases},\ p \in \mathbb{Z} \setminus \{0\}$$

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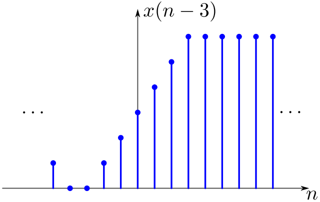

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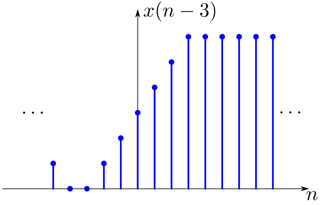

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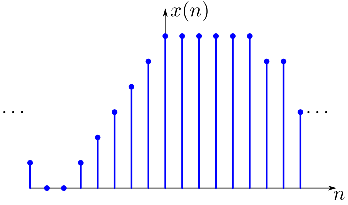

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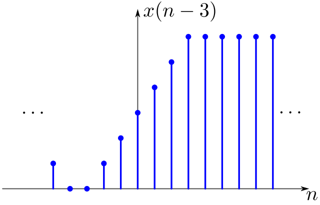

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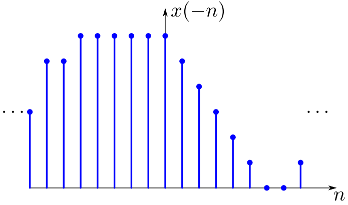

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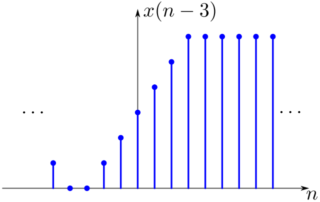

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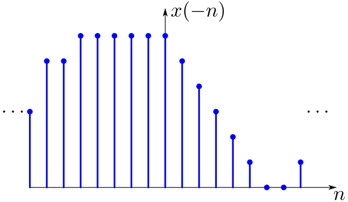

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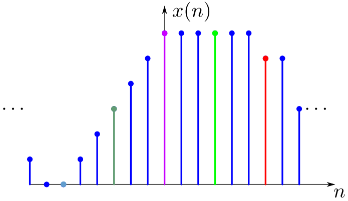

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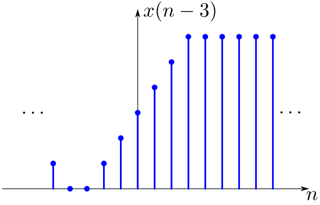

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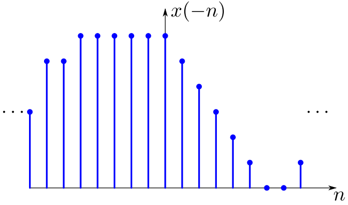

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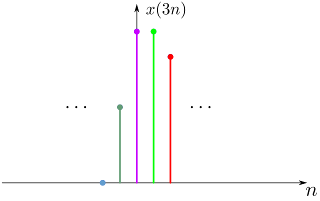

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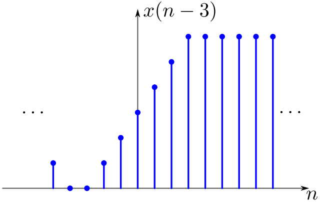

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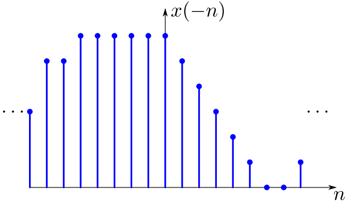

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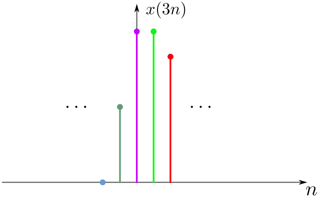

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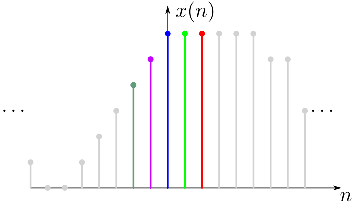

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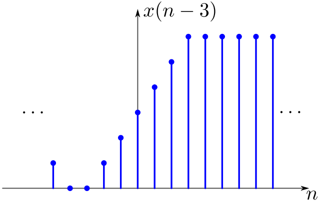

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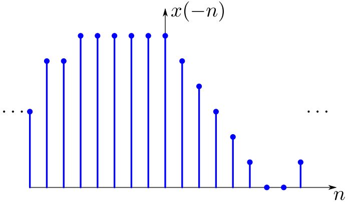

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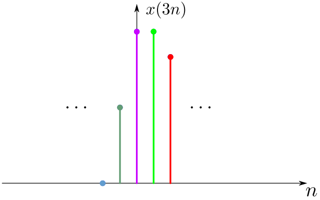

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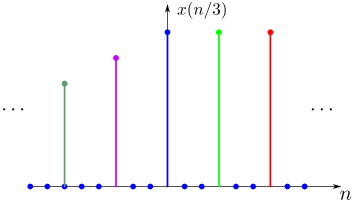
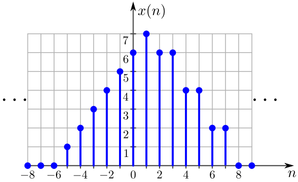


Transformations of the independent variable - II

- The order of the operations can make easier (or harder) the calculation of the modified signal.
  - Let us consider the following signal: $x(3-2n)$



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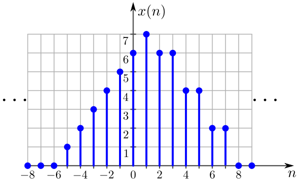

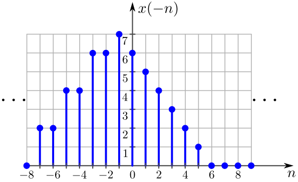

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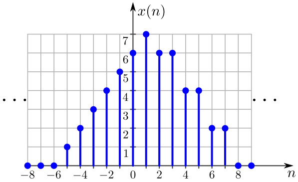

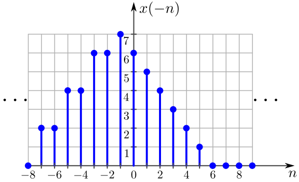

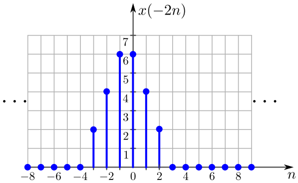

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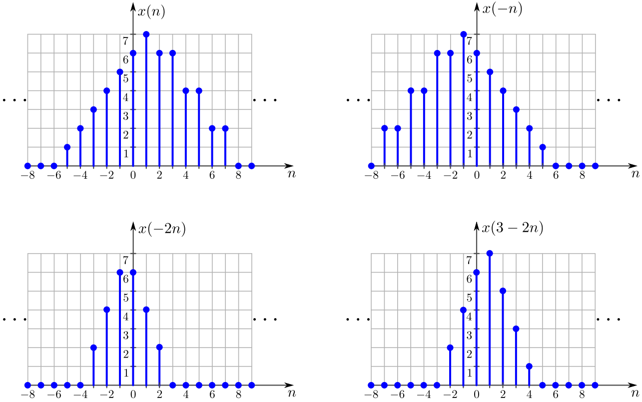

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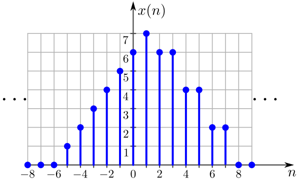
- In general, it is easier to postpone the decimation or zero-padding to the last step.



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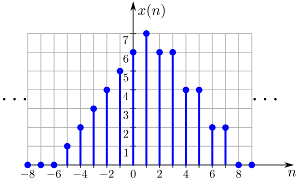

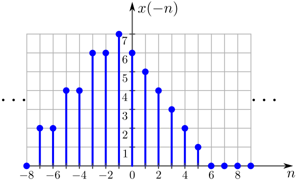

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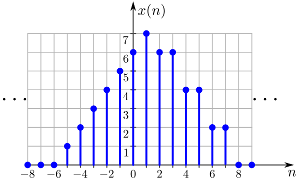

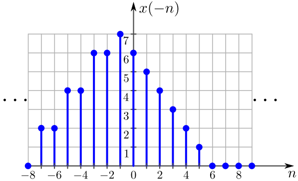

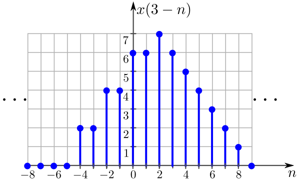

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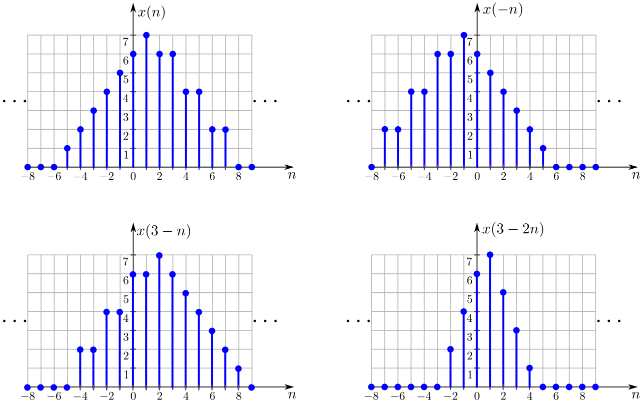

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- It plays the same role as the unit impulse function (Dirac delta function).
- It is defined as

$$\boxed{\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}}$$

- Regarding the convolution, it meets an identity analogous to the one for CV signals:

$$x(n) * \delta(n - n_0) = x(n - n_0)$$

- For this reason, a DV system with impulse response $\delta(n)$ is referred to as identity system.

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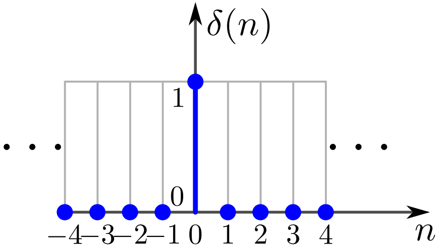

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Unit sample sequence

The unit sample sequence is often referred to as discrete-variable impulse or, simply, impulse.

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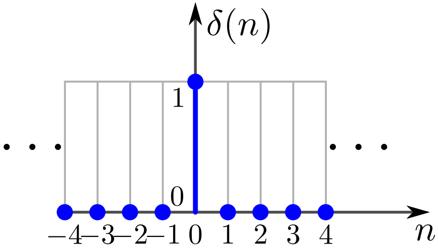

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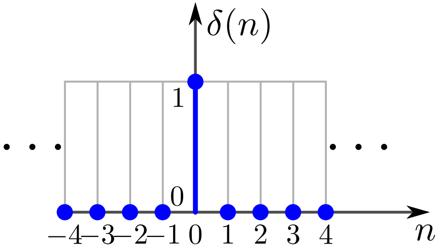

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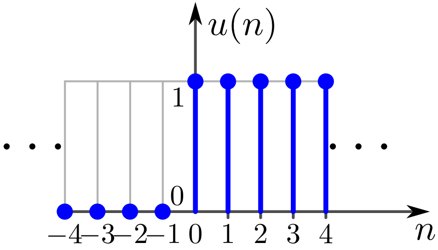

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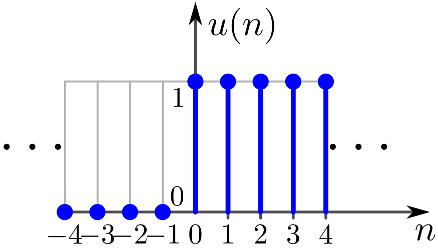

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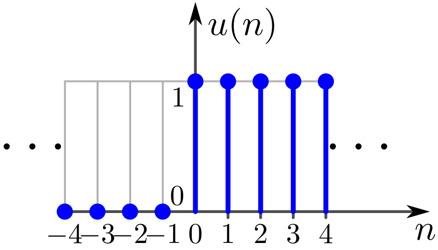

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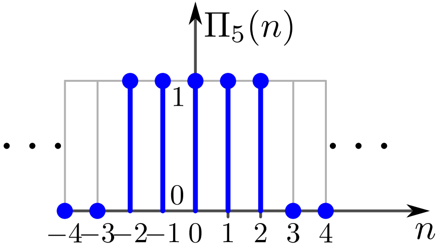

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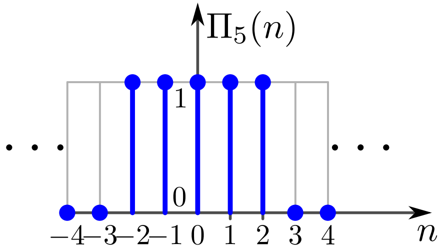

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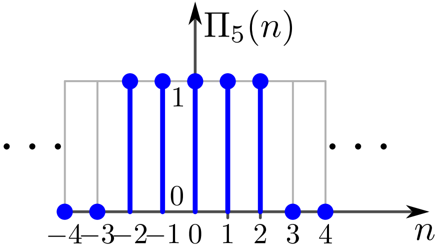

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Periodic DV signals

- If the properties of a signal are repeated in regular intervals, the signal is said to be periodic.
- That condition is mathematically expressed as:

$$x_0(n) = x_0(n + N_0), \forall n$$

- Note that a signal, whose period is N_0, is also a signal with period mN_0.
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Solution

$$e^{j\Omega_0 \mathrm{n}} = e^{j\Omega_0(n+N_0)} \Longleftrightarrow \Omega_0 N_0 = 2\pi k \quad k \in \mathbb{Z}$$

$$\boxed{\Omega_0 = \frac{2\pi k}{N_0}}$$

Note that increasing the frequency does not always result in a shorter period. In fact, it can increase it.

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Determine whether or not the following signals are periodic. If so, find the fundamental period.

1. $\exp\left(j2\pi n\right)$
2. $\exp\left(j\pi n\right)$
3. $\exp(j0.2n)$
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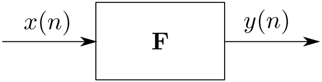
5. $\cos\left(\frac{2}{5}2\pi n\right) + \sin\left(\frac{2}{3}2\pi n\right)$ Yes, the period of each signal is 5 and 3, and, therefore, the common period is the l.c.m of both $N_0 = 15$

Discrete-variable systems and related properties

- The DV system, as in the CV case, are characterized by an operator



- These systems can also meet (or not) the following properties: linearity, shift-invariance, memoryless, invertibility, causality and stability.
- All the previous properties are defined exactly as the ones for the CV case.

Discrete-variable linear systems

- If the system is linear, then

$$y(n) = \mathbf{F}\left(x(n)\right) = \mathbf{F}\left(x(n) * \delta(n)\right) = \mathbf{F}\left(\sum_{n'=-\infty}^{\infty} x(n')\delta(n-n')\right) =$$

$$= \sum_{n'=-\infty}^{\infty} x(n')\mathbf{F}\left(\delta(n-n')\right) = \sum_{n'=-\infty}^{\infty} x(n')h(n;n')$$

- Thus, the collection of impulse responses $h(n;n')$ completely defines the VD linear system.
- If the system is also shift-invariant, then $h(n;n') = h(n-n')$ yielding:

$$\boxed{y(n) = x(n) * h(n)}$$

- Thus, the operator **F** of a LSI system is equivalent to a convolution with the impulse response of the system $h(n)$.

Discrete-variable linear systems

Properties of DV LSI systems

The properties of the LSI systems can be found from the impulse responses exactly as in the case of CV LSI systems:

$$\begin{aligned}
\text{LSI system memoryless} &\Leftrightarrow h(n) = K\delta(n) \\
\text{LSI system causal} &\Leftrightarrow h(n) = 0, \forall n < 0 \\
\text{LSI system invertible} &\Leftrightarrow \exists h^{-1}(n) \text{ such that } h(n) * h^{-1}(n) = \delta(n) \\
\text{LSI system stable} &\Leftrightarrow \sum_{n=-\infty}^{\infty} |h(n)| < \infty
\end{aligned}$$

Discrete-variable linear systems

Exercise

Prove the previous properties.