STATISTICAL METHODS FOR BUSINESS.
First assessment test (Probability Calculus). SOLUTIONS

1. Twenty cars are running in a car race. The number of ways of assigning the three medals (gold, silver, bronze) among the pilots is

- ❑ 57.
- ❑ 6,840. **(Right)**
- ❑ 8,000.

2. If events A and B are independent then:

- ❑ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- ❑ $P(A \cap B) = P(A)\ P(B)$.
- ❑ Both answers are true. **(Right)**

3. If $Y = 4 - 3X$, with X being a random variable, then:

- ❑ $E(Y) = 4 - 3\,E(X)$. **(Right)**
- ❑ $Var(Y) = -9Var(X)$.
- ❑ Both answers.

4. The integral of the density function of a continuous random variable is:

- ❑ The probability function.
- ❑ The distribution function. **(Right)**
- ❑ None of the answers.

5. Given two independent random variables, namely $X \sim N(0, 3)$ and $Y \sim N(1, 4)$, the variable $X + Y$:

- ❑ Has N(1, 7) distribution.
- ❑ Has N(1, 5) distribution. **(Right)**
- ❑ Is not always normally distributed.

Time limit: 10 MINUTES

Scoring criteria	Right answer: +2 point
	Wrong answer: -1/2 point

STATISTICAL METHODS FOR BUSINESS.
Second assessment test (Statistical Inference). SOLUTIONS

1. Given a simple random simple (s.r.s.) drawn from a normal population, the elements of the sample are:
   - ❑ Normal random variables.
   - ❑ Independient random variables.
   - ❑ Both answers. (**Right**)

2. In order to estimate a certain parameter, two unbiased estimators, Ta and Tb, are suggested. Knowing that Var(Ta)>Var(Tb), it could be stated that:
   - ❑ Tb is more efficient than Ta.
   - ❑ The mean square error of Tb is smaller than that of Ta.
   - ❑ Both answers. (**Right**)

3. X is a random variable having a normal distribution with known σ. Given a simple random sample of size 35, the confidence interval for the population mean would be obtained based on:
   - ❑ Student's t distribution with 35 d.f.
   - ❑ the N(0,1) distribution. (**Right**)
   - ❑ Studentt's t distribution with 34 d.f.

4. In order to estimate the population proportion (p) with given confidence level and error margin, the minimum simple size can be obtained by replacing p(1-p) with its least upper bound, which is reached when:
   - ❑ p=0.5. (**Right**)
   - ❑ p=0.2.
   - ❑ p=0.8.

5. When testing the null hypothesis H_0: $\mu = \mu_0$ versus H_0: $\mu > \mu_0$ the null is rejected if:
   - ❑ The sample mean is smaller than μ_0.
   - ❑ The p-value is close to zero. (**Right**)
   - ❑ Both answers.

Time limit: 10 MINUTES

Scoring criteria	Right answer: +2 point
	Wrong answer: -1/2 point

Facultad de Economía y Empresa
Grado en Administración y Dirección de Empresas
STATISTICAL METHODS FOR BUSINESS
Convocatoria de Mayo. 28/05/2012

1. Let X and Y be independent random variables:
a) Deduce the abbreviated formula for the variance of X.
b) Show that the variance of X + Y is the sum of the variances of the two summands.

2. Derive the expression for the pivotal statistic used to carry out inferences on the population mean, given a simple random sample of size *n* drawn from a normal population with unknown variance. Justify the probability distribution of the pivot.

Solutions to questions 1 and 2 can be found in any of the reference books for the course.

3. Let X be a N(μ, σ) population from which a simple random sample of size 5 is drawn, (X_1, X_2, X_3, X_4, X_5).
a) What are the basic characteristics of the variables in the sample? What probability distribution do they have?
Independent and identically distributed normal random variables
b) The following estimators are proposed in order to estimate μ:

$$T_1 = \overline{X}_5 \qquad T_2 = \frac{X_1 + 2X_3 + X_4 + X_5}{5}$$

Given $MSE_{T_1}(\mu) = \frac{\sigma^2}{5}$, analyze which of the two estimators is more efficient.

T_1 is more efficient because $MSE_{T_1}(\mu) = \frac{\sigma^2}{5} < MSE_{T2}(\mu) = \frac{7\sigma^2}{25}$

4. The owner of a gift shop located in a thematic park estimates that 30% of park visitors come to his store.
a) Calculate the probability that the third visitor to the park is the first to go to the store.
Geometric model, P(X=3)=0.147
b) 200 visitors have come to the park one morning. What is the probability that at least 70 of them go to the store?
Central Limit Theorem N(0.30*200, square root(0.30*200*0.7)
P(X>=70)=0.05
c) Customers' expenditures (X, in euros) of each customer of the store is a random variable with expectation 6 and standard deviation 1.5. Calculate the probability that the spending of one customer deviates from its expected value in less than 2 euros.
Chebyshev inequality. Probability is at least 0.4375

5. The owners of a gym want to study the time devoted to sports by the young people living in the neighbourhood, which is a random variable (X, in hours per week) with variance 4.25.
a) Calculate the minimum sample size required to estimate the expected weekly sport time with 95% confidence level and an error margin of 0.4 hours. **n=103**
b) An average of 3 hours per week has been obtained for a random sample of 36 young people from the neighbourhood. Compute a 90% confidence interval for the expectation of X. **(2.32-3.67)**

6. The manager of a hospital is analyzing the monthly expenditure on food (X, in hundred thousand €). A simple random sample of 16 months has been selected for this purpose.

a) The above sample has asymmetry coefficient $g_1 = 0.32$ and kurtosis coefficient $g_2 = -0.52$. Test for the normality of X.

d*=0.45, p=0.8 The null hypothesis about the normality of X cannot be rejected.

b) The hospital manager assures that the variance of food expenditure does not exceed 3.5. If a variance of 4.02 was obtained in the above sample, is this opinion well founded?

d*=17.2, p=0.3 It cannot be rejected that the variance of food expenditure does not exceed 3.5.

c) The manager also states that the percentage of patients satisfied with the food in the hospital is 75%. In a random sample of 80 patients, 48 of them said they are satisfied with the food. Does the sample support manager's claim?

d*= -3.1, p=0.000 It can be rejected that the percentage of patients satisfied with the food in the hospital is 75%