



Unit 2: Random variables

A. Class exercises

DEMAND The demand for a certain item is a random variable distributed as follows:

Number of units demanded	1	2	3	4
Probability	0.25	0.45	0.15	0.15

- Is this a probability function?
- Obtain the distribution function. What is the probability that demand is less than 4 units?
- Calculate and interpret the expected value and variance of the variable.

[Adapted from PERALTA, MJ and others (2000): **Estadística Descriptiva y nociones de Probabilidad. Estadística I. Probabilidad. Ed. Thomson. Ejercicio 7.7** pág. 267]

FOOTBALL The variable X below shows the number of goals that a football team scores in each game, giving the following information on the number of goals scored in the last 100 games.

Number of goals	Number of matches
0	40
1	30
2	15
3	10
4	5

- Obtain the probability distribution of the variable X "number of goals scored per game."
- What is the expected number of goals per game? Calculate the dispersion of the number of goals.
- A bonus of 600 euros is granted to the team for each goal scored. Obtain the distribution of the variable "income of the workforce by goals scored in a game."
- What is the probability that the players get more than 1,200 euros in a match? Get the expected amount of bonuses for goals scored in a match.

STORES The daily expenditure (X, in euros) spent by a certain customer in a department store is distributed according to the following density function:

$$f(x) = x^2 / 9000, \text{ if } 0 < x < 30,$$
$$f(x) = 0 \quad \text{otherwise.}$$

- Calculate the probability that the client has a daily spending between 15 and 20 euros.
- What is his/her expected daily expenditure? Calculate the variance of spending.
- Next winter the department store is granting the following "discount vouchers", whose amount depends on the client's expenditure:
 - 1 euro for spending between 10 and 20 euros,
 - 1.5 euros for spending between 20 and 25 euros,
 - 3 euros if the expense exceeds 25 euros,

Derive the distribution and the expected value of the discounts obtained by the client.

- The customer has paid for his/her purchases in the store with a credit card which applies a 4% charge. What is the probability that the customer has paid (with charges included)



between 10 and 15 euros?

SMEs The random variable X , which includes monthly R&D costs (in thousands of euros) for small and medium size firms located in a certain region, is distributed according to the following density function:

$$f(x) = ax, \text{ if } 0 < x \leq 4,$$

$$f(x) = 0 \text{ otherwise.}$$

- Determine the value of a .
- Obtain and plot the distribution function.
- What is the probability that monthly costs are higher than 2,000 euros?

CASINO A casino advertises that the expected prize won by its clients is 600 euros, with risk $\sigma = 360$.

- Calculate the probability that a player wins between 150 and 1,050 euros.
- How would the above probability change if $\sigma = 420$?
- What is the probability that the prize won by a player deviates at least 504 euros from its expectation?

[From *Análisis de Datos Económicos II. Métodos inferenciales*, exercise 2.3 pp. 103-104]

TRANSFORMATION

- Let X be a continuous random variable. Show that for any constant a and c it holds:
$$E(aX + c) = aE(X) + c.$$
- Let X be a random variable. Show that for any constants a and c it holds:

$$\text{Var}(aX + c) = a^2 \text{Var}(X).$$



B. Proposed Exercises

PROJECT An economist wishes to estimate the total cost of a project. His salary is composed of a fixed part of € 12,000 plus a variable part of € 300 per day of work. The project can be performed between 7 and 11 days, and the economist has derived the following subjective probability distribution for the random variable X: "Number of days it will take to implement the project":

X	7	8	9	10	11
P (X = x)	0.10	0.20	?	0.30	0.10

- What is the probability that the project is carried out in 9 days? Justify your answer.
- Determine the expected cost of the project and its variance. 7% of the total cost is devoted to payment of taxes and fees. Calculate the expected payment of taxes plus fees.

[Adapted from ESTEBAN, J. et. al. (2004): *Estadística Descriptiva y nociones de Probabilidad. Estadística I. Probabilidad*. Ed. Thomson. Exercise 7.7 pg. 267]

RESULTS. a) $P(X = 9) = 0.30$. b) $E(Y) = 14,730$, $Var(Y) = 116,100$. c) $E(Z) = 1031.1$.

PRODUCTION The production volume of a company is a random variable (X) whose density function is

$$f(x) = ax, \text{ if } 0 < x \leq 5,$$

$$f(x) = 0 \text{ otherwise.}$$

- Determine the value of a.
- Obtain the distribution function of this model.
- The total costs of the firm (C, in thousand euros) is computed as $C = 3 + 0.18 X$. Derive the probability distribution of the costs.

RESULTS.

a) $a = 0.08$.

b)

$$F(x) = 0 \quad \text{if } x < 0,$$

$$F(x) = 0.04x^2 \quad \text{if } 0 \leq x < 5,$$

$$F(x) = 1 \quad \text{if } x \geq 5.$$

c)

$$F_C(c) = 0 \quad \text{if } c < 3,$$

$$F_C(c) = 0.04[(c - 3) / 0.18]^2 \quad \text{if } 3 \leq c < 3.9,$$

$$F_C(c) = 1 \quad \text{if } c \geq 3.9.$$

INCOME The annual revenue (X, in thousands of euros) of a certain woman is distributed according to the following density function:

$$f(x) = \begin{cases} \frac{120-x}{2400} & \text{if } 40 < x < 80, \\ 0 & \text{otherwise.} \end{cases}$$



- a) What is the probability that revenues exceed 60,000 euros? And that they range between 50,000 and 70,000 euros?
- b) Calculate the expected annual income and the variance of income.
- c) This person annually donates to charities a sum which depends on her income:
- if income is less than 50,000 euros, 1,500 euros are donated,
 - if income is between 50,000 and 65,000 euros, 2,500 euros,
 - if income exceeds 65,000 euros, the donation is 5,000 euros.

Derive the probability distribution of the donations. What is the expected donation?

RESULTS. a) 0.5. b) $E(X) = 57.8$; $\text{Var}(X) = 125.9$.

c)

y_i	$P(Y = y_i)$
1,500	0.3125
2,500	0.3906
5,000	0.2969

$E(Y) = 2929.75$.

CARS A family is analyzing the gasoline consumption of their car (X , in liters). The expected consumption is $\mu = 160$ liters per month, with standard deviation $\sigma = 8$.

- a. Calculate the probability that a given month the gas consumption of the car differs from its expectation in less than 10 liters.
- b. Due to misuse of the car its monthly consumption has increased by 10 liters. Justify how this affects the expectation and the variance of gasoline consumption.

RESULTS. a) $P(|X - \mu| < 10) \geq 0.36$. b) $E(Y) = 170$, $V(Y) = 64$.