UNIT 4: CONTINUOUS PROBABILITY MODELS

4.1.- Uniform Distribution
4.2.- Normal Distribution
4.3.- Other continuous distributions

UNIT 4. GOALS

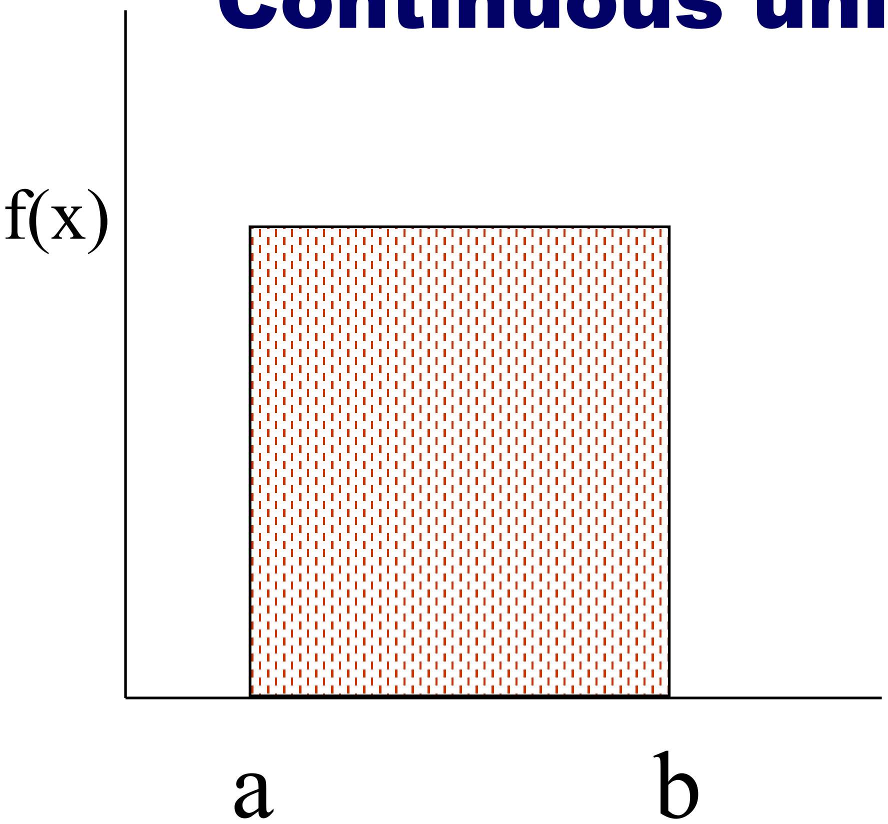
- To identify the uniform distribution and calculate probabilities.

- To describe the characteristics of the normal distribution and apply the standarization process.

- To use the normal distribution tables in order to calculate probabilities or obtain values.

- To identify other continuous probability distributions.

Continuous uniform distribution U(a,b)



Density function

$$f(x)=k=\frac{1}{b-a} \qquad a<x<b,$$

$$f(x)=0 \qquad \text{otherwise}$$

Distribution function

$$F(X)=\begin{cases} 0 & x<a \\ \dfrac{x-a}{b-a} & a\le x<b \\ 1 & x\ge b \end{cases}$$

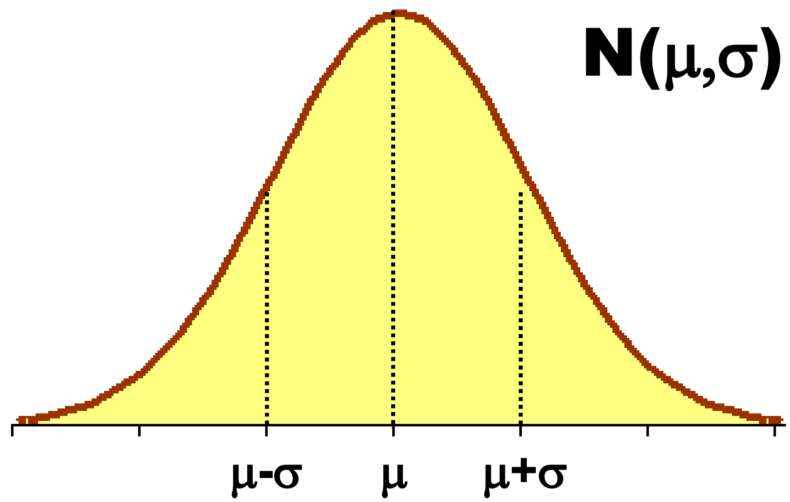
Expected value

$$\mu=\frac{a+b}{2}$$

Variance

$$\sigma^2=\frac{(b-a)^2}{12}$$

Normal (or Gaussian) Distribution

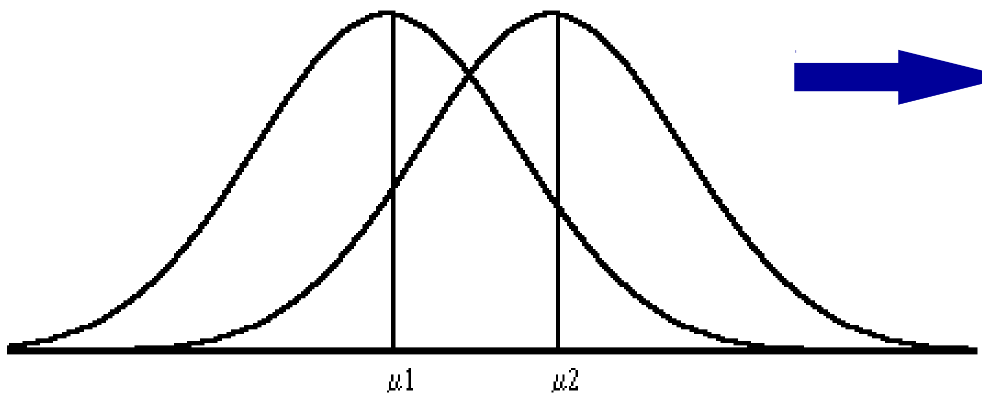


Density function

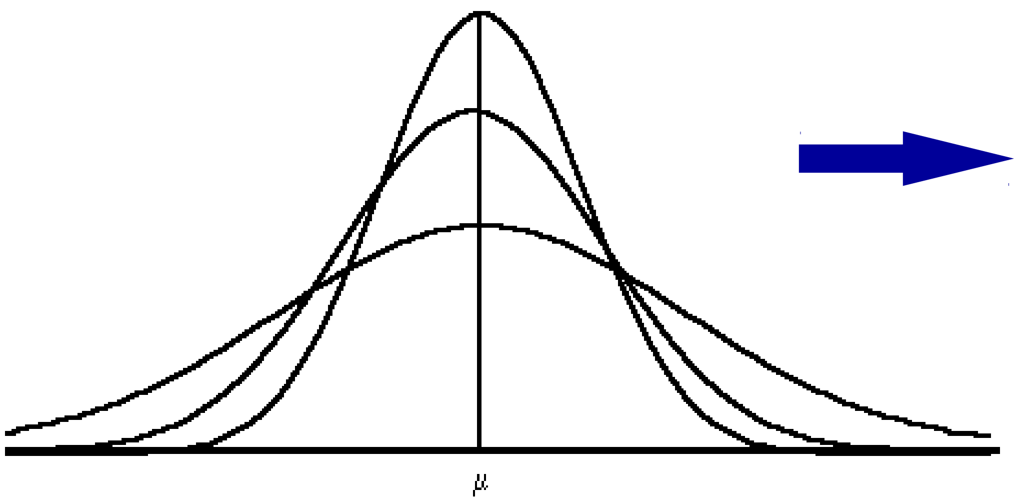
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < +\infty$$

Expected value $E(X) = \mu$ **Variance** $Var(X) = \sigma^2$

Normal Distribution N(μ,σ)

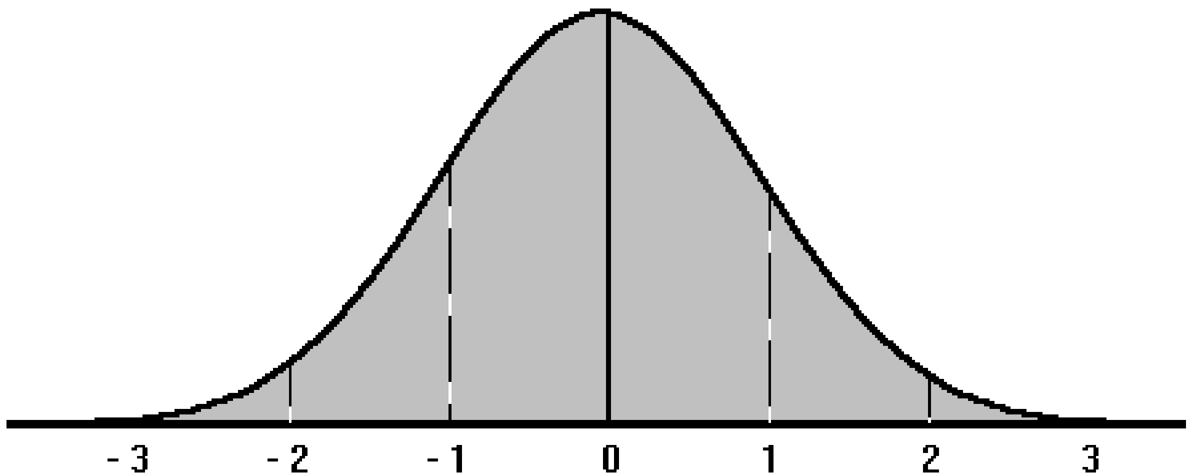


Parameter μ indicates the center of the distribution



Parameter σ indicates the "spread" of the distribution

Standard Normal Distribution N(0,1)



Density function

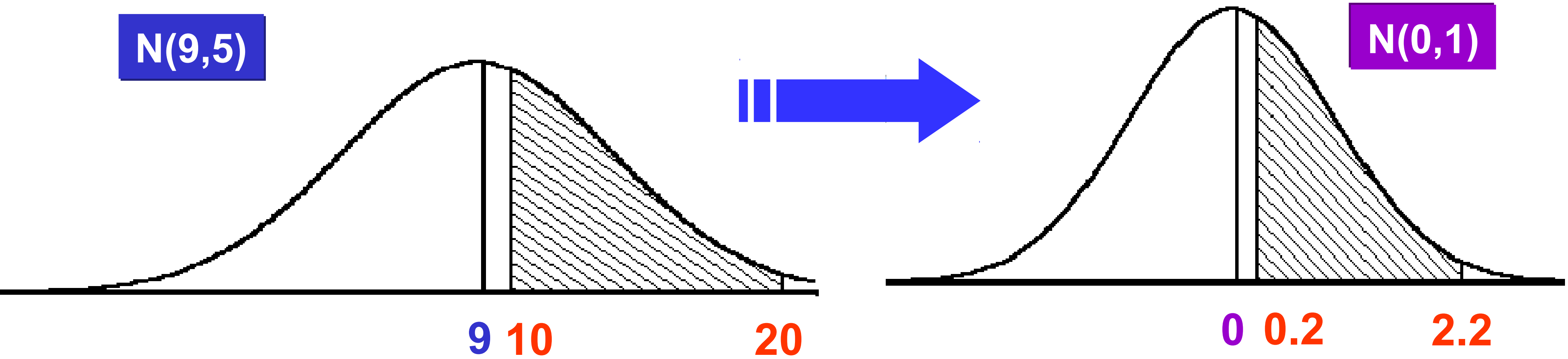
$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}};-\infty<x<+\infty$$

Distribution function

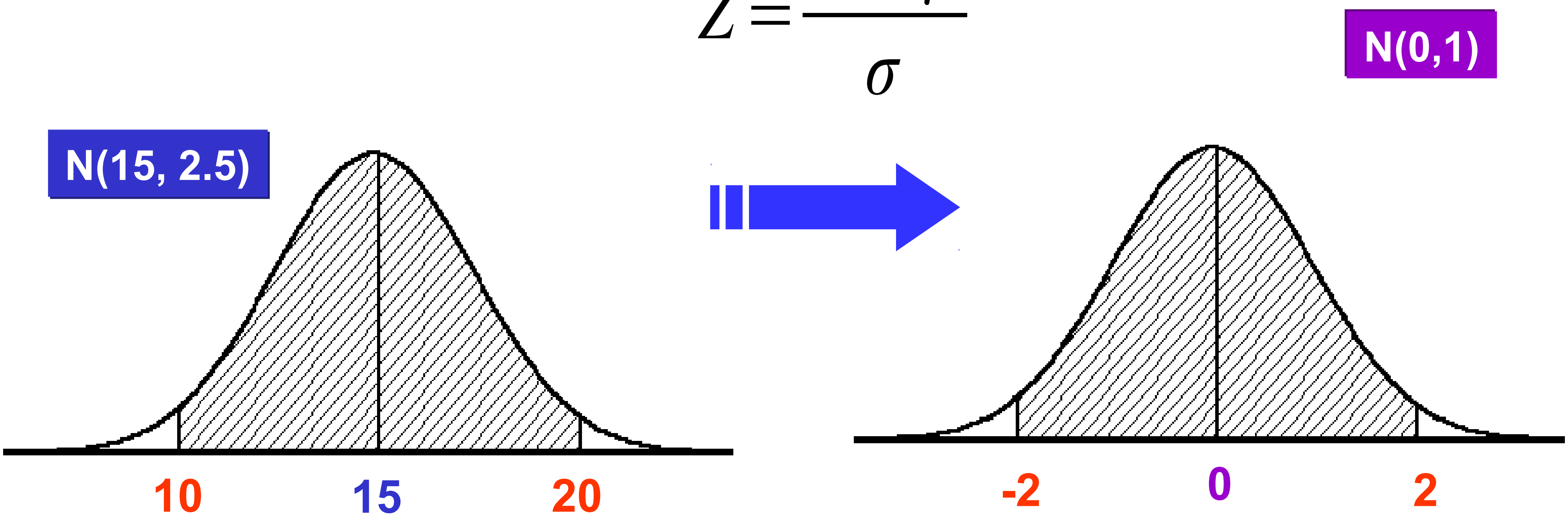
$$F(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$$

Expected value E(X)=0 **Variance** Var(X)=1

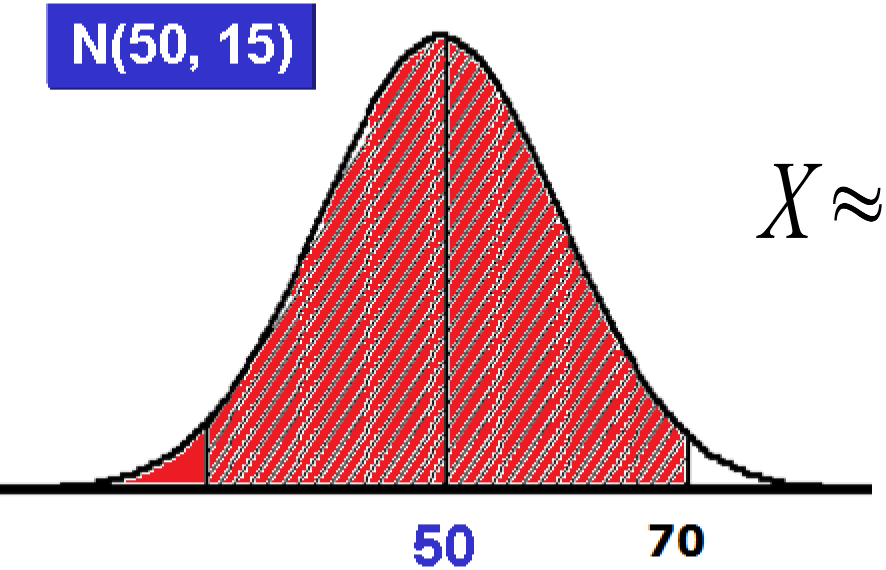
Standardization process



$$Z = \frac{X - \mu}{\sigma}$$



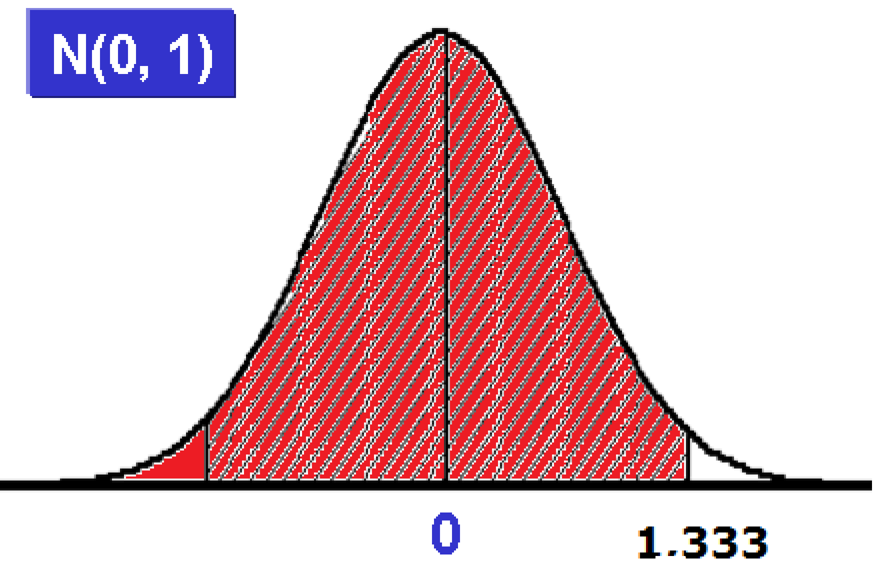
Calculating probabilities



$$X \approx N(50, 15) \qquad \mathrm{P}(X \leq 70)$$

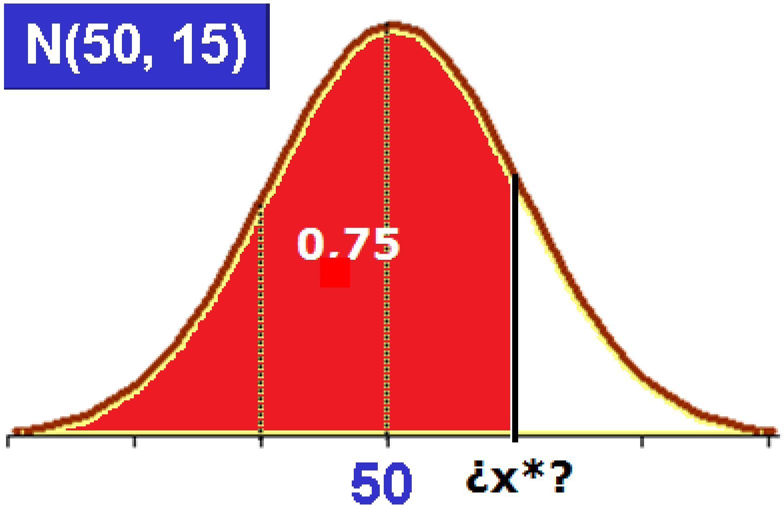
Standardization

$$\frac{X-50}{15} \approx N(0,1) \qquad P(Z \leq 1.3)$$



$$P(X \leq 70) = P\left(\frac{X-50}{15} \leq \frac{70-50}{15}\right) = p(Z \leq 1.3) = 0.9088$$

Obtaning values



Finding a value x* whose cumulative probability is 75%

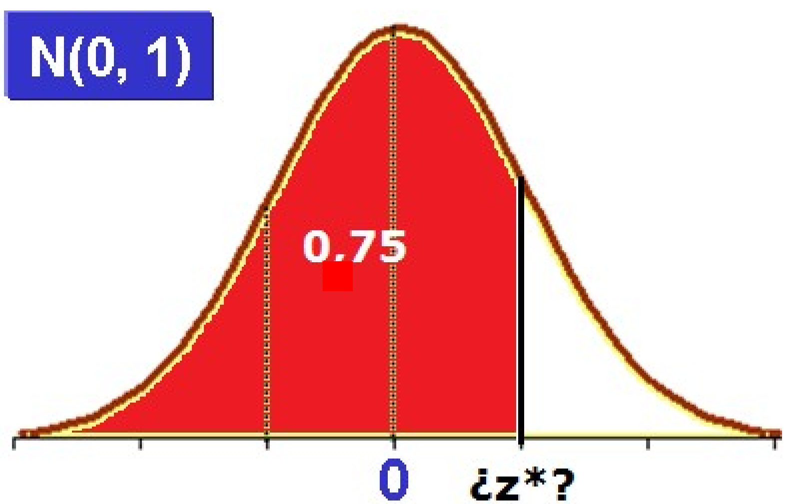
$$Z = \frac{X - 50}{15} \approx N(0,1)$$

$$P(X \le x^*) = P\left(\frac{X-50}{15} \le \frac{x^*-50}{15}\right) =$$

$$= P(Z \le z^*) = 0.75$$

$$z^* = 0.6743 \Rightarrow \frac{x^* - 50}{15} = 0.6743$$

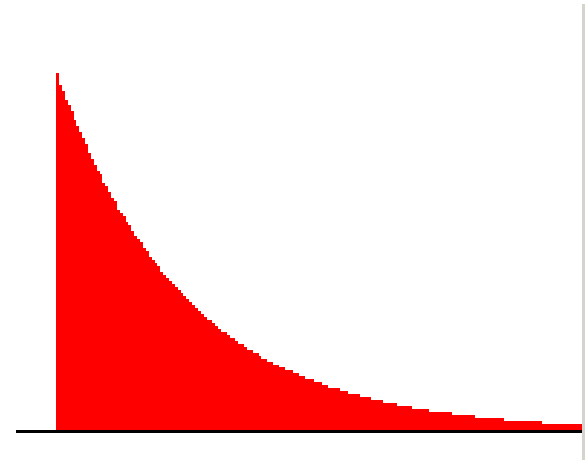
$$x = 60.114$$



Exponential distribution

X: "Waiting time for an event to occur"

$X \approx Exp(\alpha) \qquad \alpha>0$



Density function

$$f(x) = \alpha\, e^{-\alpha x} \qquad x>0,\ \alpha>0$$

Characteristics

$$E(X)=1/\alpha$$

$$Var(X)=1/\alpha^2$$