UNIT 9 .- INTRODUCTION TO HYPOTHESIS TESTING.

UNIT 9. GOALS

- State a statistical hypothesis and distinguish the types of errors in hypothesis testing.
- Interpret the meaning of p-values.

STATEMENT OF HYPOTHESES

There are constant returns to scale

The expected demand is at least 30000 units

75% of Europeans are in favour of the EU enlargement

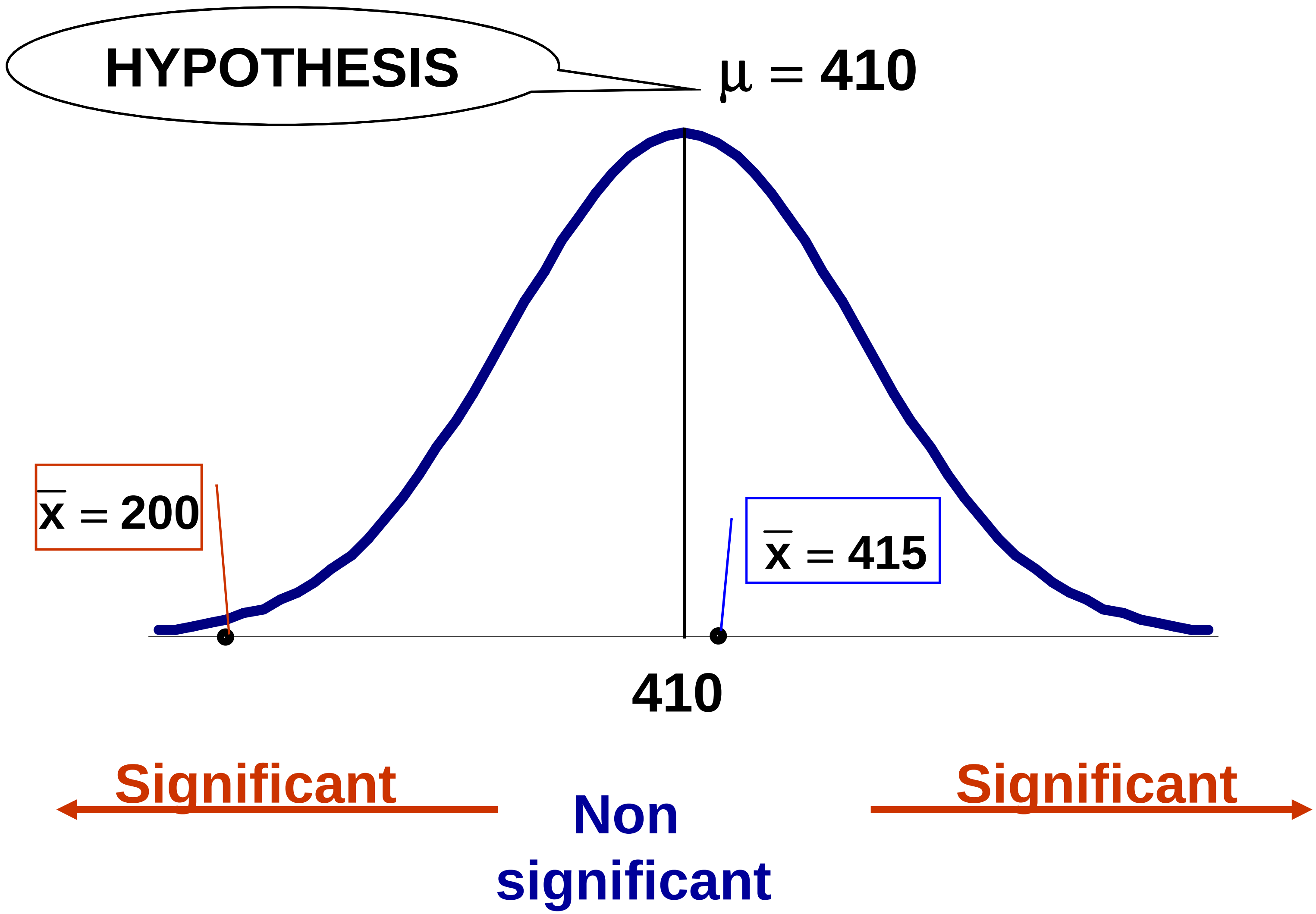
The weight follows a normal distribution

Hypothesis testing involves three types of information:

- That related to the hypothesis to test
- Information about the population
- Sample information

EXAMPLE

The monthly average prodution of certain mineral is 410 T. Two samples are drawn, obtaining the following results:



HYPOTHESIS TESTING

- State the hypothesis
- Determine the right statistic to use in order to performance the test
- Decide whether to reject or not the hypothesis

HYPOTHESIS TESTING

The essential idea in hypothesis testing is to analyse the differences between the hypothesis and sample information

HYPOTHESIS

SAMPLE

Are the significant diferences between the hypothesis and the sample?

YES	**NO**
Low probability of observing such differences	**High probability of observing such differences**
REJECT	**FAIL TO REJECT**

Types of errors in Hypothesis Testing

Two types of errors can be distinguished in hypothesis testing:

Is the hypothesis H_0 true?

Is H_0 rejected?

	YES	NO
YES	TYPE I ERROR	Right decision
NO	Right decision	TYPE II ERROR

Types of errors in Hypothesis Testing

<u>Example</u>: In a criminal trial, a defendant is considered not guilty as long as his or her guilt is not proven.

Then, the hypothesis is : The defendant is not guilty

Is the hypothesis true?

Is the hypothesis of innocence rejected?

	YES	NO
YES	**Convinct an Innocent person**	Convict a guilty person
NO	Free an innocent person	**Free a guilty person**

Probabilities of Type I and Type II errors

Is the hypothesis H_0 true?

Is H_0 rejected?

	YES	NO
YES	**P(ERROR I) = α** **Significance Level**	**1 - β** **Power**
NO	**1 - α** **Confidence**	**P(ERROR II) = β**

Stages in hypothesis testing

There are three basic stages in the hypothesis testing process:

1. State the null and alternative hypotheses

2. Carry out the test:

2.1. Classical approach

2.2. P-value approach

3. Conclusion:

Reject or fail to reject the null hypothesis

Stating the hypotheses

The hypothesis:

- is a research assumption
- can be tested using sample information

H_0: NULL HYPOTHESIS

The assumption to test

H_1: ALTERNATIVE HYPOTHESIS

Complementary statement

TYPES OF HYPOTHESES

SIMPLE $\theta = \theta_0$

COMPOSITE $\theta \neq \theta_0$ Two-tailed

$\theta \leq \theta_0$ $\theta \geq \theta_0$ One-tailed

Approaches to hypothesis testing

CLASSICAL APPROACH

- Given a certain significance level α (10%, 5%, 1%)
- Determine the threshold value of the test statistic from which the hypothesis is rejected (**Critical value**)
- If the test statistic goes over such threshold, then the the null hypothesis is **rejected**

P-VALUE APPROACH

- Obtain the test statistic **d***
- Obtain the **p-value** which is the probability of obtaining a test statistic at least as extreme as d*
- If the **p-value is small**, then the result is statistically significant and the null hypothesis is **rejected**

When can we say that a result is statistically significant?

-The p-value is a probability that is related to how likely is to get a certain sample result, under the assumption that the null hypothesis is true

- The smaller the p-value, the higher the evidence to reject the null hypothesis

- The following threshold values are considered:

$\alpha = 10\%$ $\alpha = 5\%$ $\alpha = 1\%$

Implementation of hypothesis testing

$$\boxed{\begin{array}{l} H_0: \theta \leq \theta_0 \\ H_1: \theta > \theta_0 \end{array}}$$

One-sided test

CLASSICAL APPROACH

- Given a certain significance level α

  (10%, 5%, 1%)

- Obtain the critical value k

$$P(d > k/H_0) = \alpha$$

- Determine the critical region

k Reject

P-VALUE APPROACH

- Obtain the test statistic d*

- Obtain the p-value

$$p = P(d > d^* / H_0)$$

(rigth tail)

- Reject if p is small

  (the result is statistically significant)

Implementation of hypothesis testing

$$\boxed{\begin{array}{l} H_0 : \theta = \theta_0 \\ H_1 : \theta \neq \theta_0 \end{array}}$$

Two-sided test, symmetric distribution

CLASSICAL APPROACH

- Given a certain significance level α (10%, 5%, 1%)
- Obtain the critical values k_1 & k_2

$$P(d < k_1 / H_0) = P(d > k_2 / H_0) = \frac{\alpha}{2}$$

- Determine the critical regions

P-VALUE APPROACH

- Obtain the test statistic d*
- Obtain the p-value

$$p = P(|d| > |d^*| / H_0)$$

(symmetric distributions)

- Reject if p is small

(the result is statistically significant)