Mathematics and Associated Teaching Methods II

GEOMETRY

VOLUME AND AREA (3D)

Degree in Primary Education Teaching

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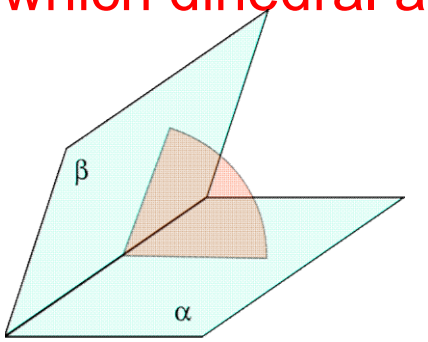
Basic Definitions

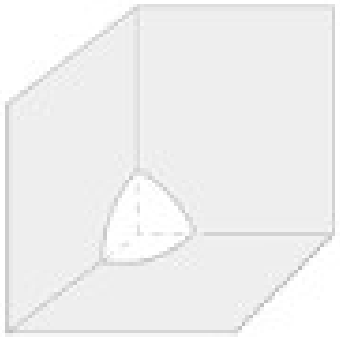
- Polyhedron: It is a solid figure bounded by polygons.
- Elements:
  - Faces (f)
  - Edges (e)
  - Vertices (v)
  - Diagonals (d)

Euler formula f+v=e+2

Regular polyhedron

Those polyhedron with faces that are equal regular polygons and which dihedral and polyhedral angles are equal.





To find how many regular polyhedron there are, we should pay attention to:

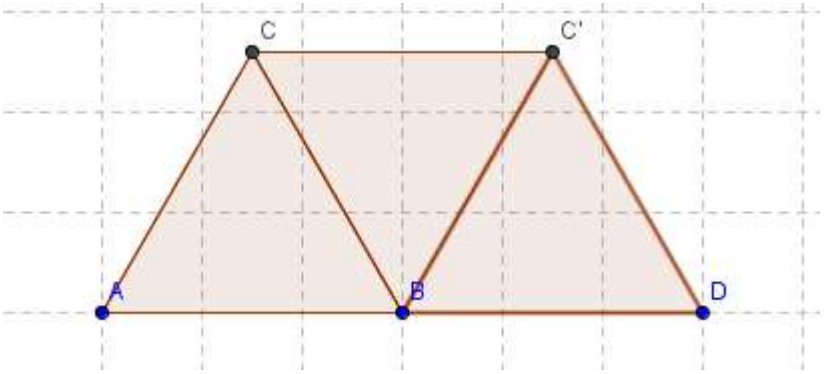
-The faces: all faces are equal.

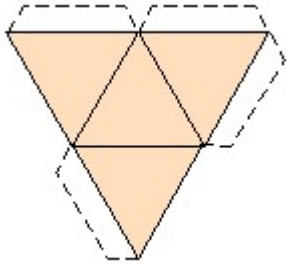
-Angles: All the angles of the faces in the same vertex should sum less than 360º.

Regular polyhedron

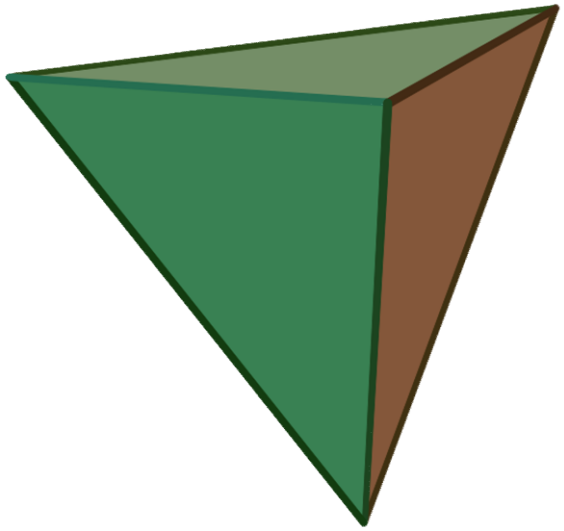
Let’s start with the equilateral triangle:

Three equilateral triangles in each vertex:



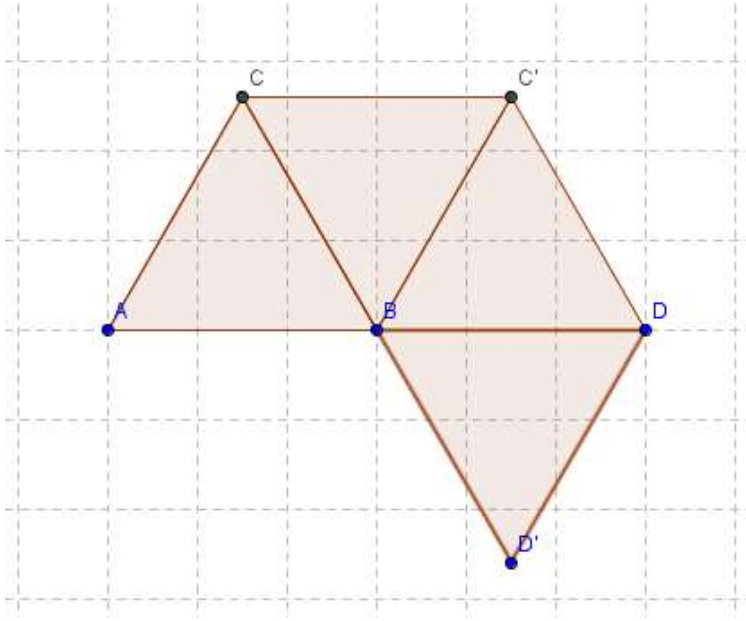


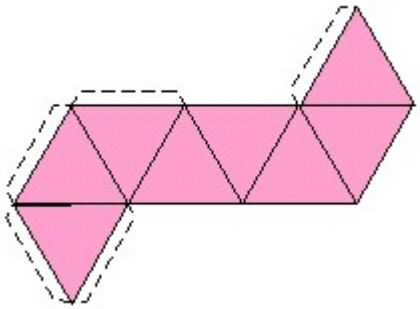
Tetrahedron

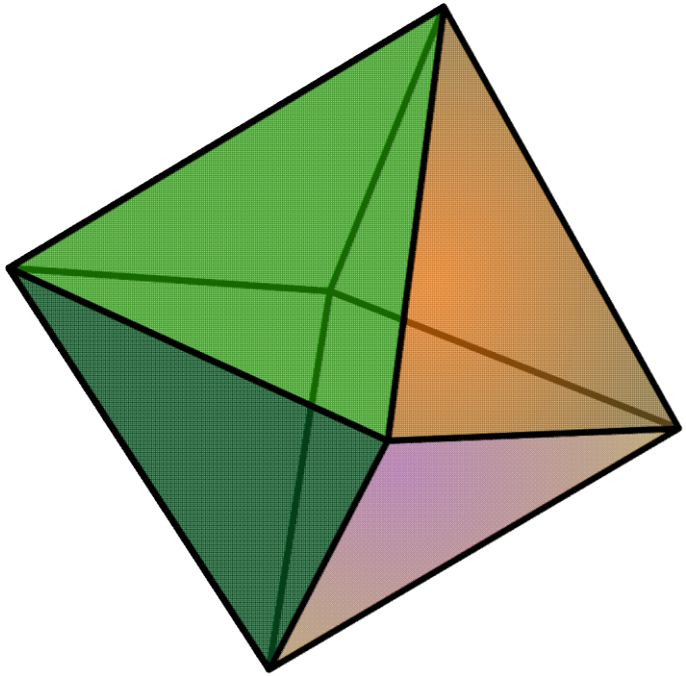


Regular polyhedron

Four equilateral triangles in each vertex:



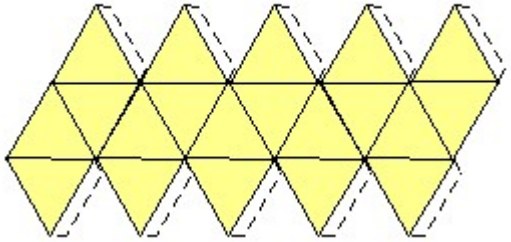


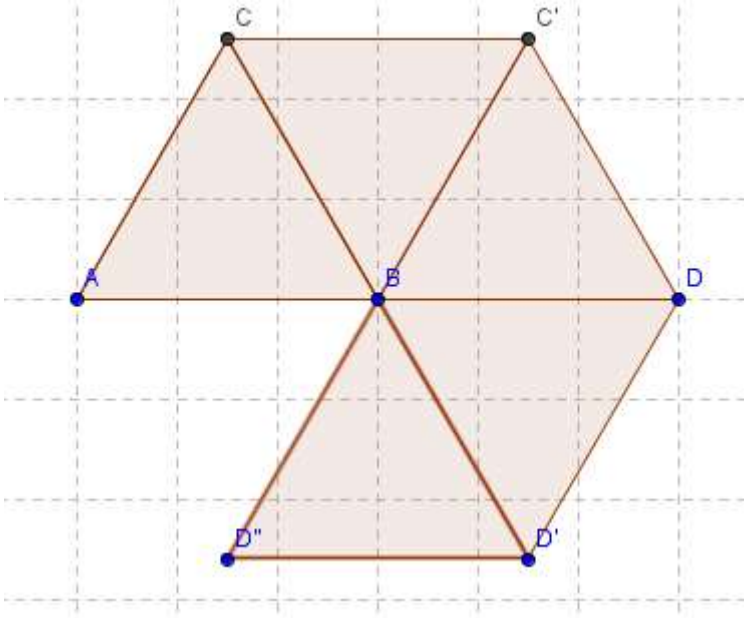


Octahedron

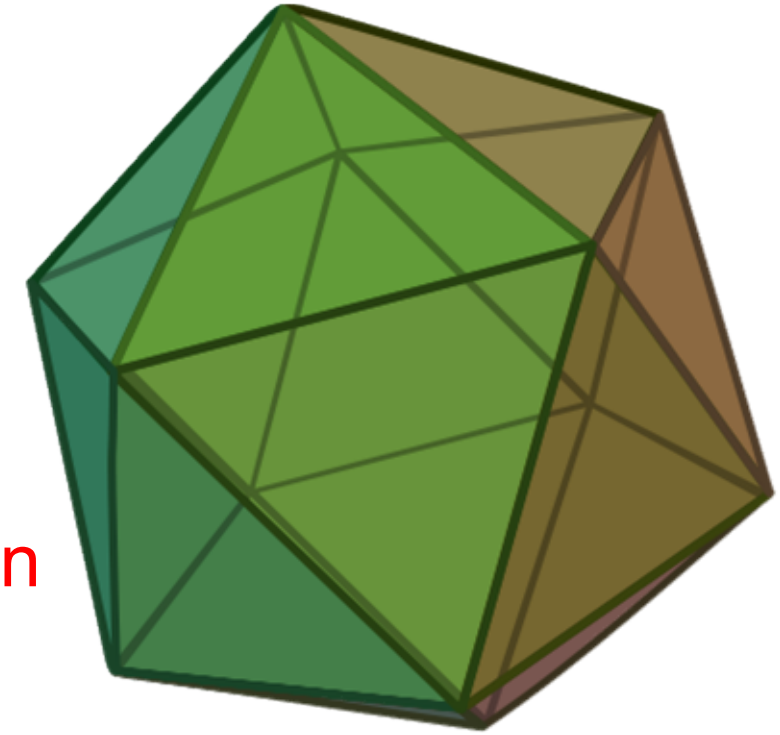
Regular polyhedron

- Five equilateral triangles in each vertex:



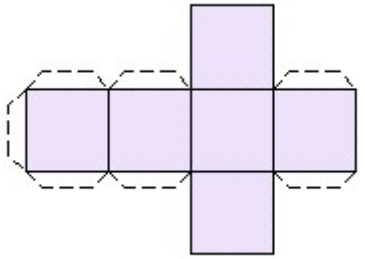


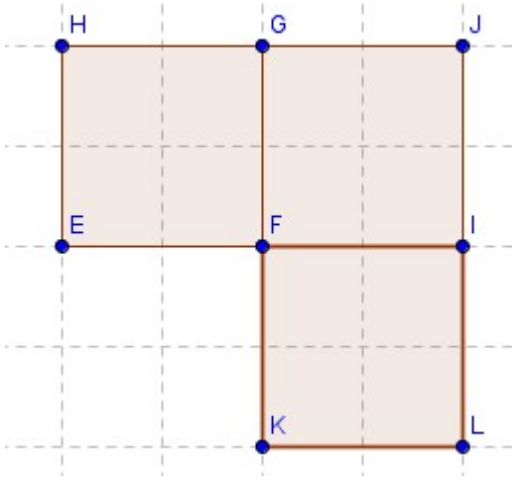
- Icosahedron



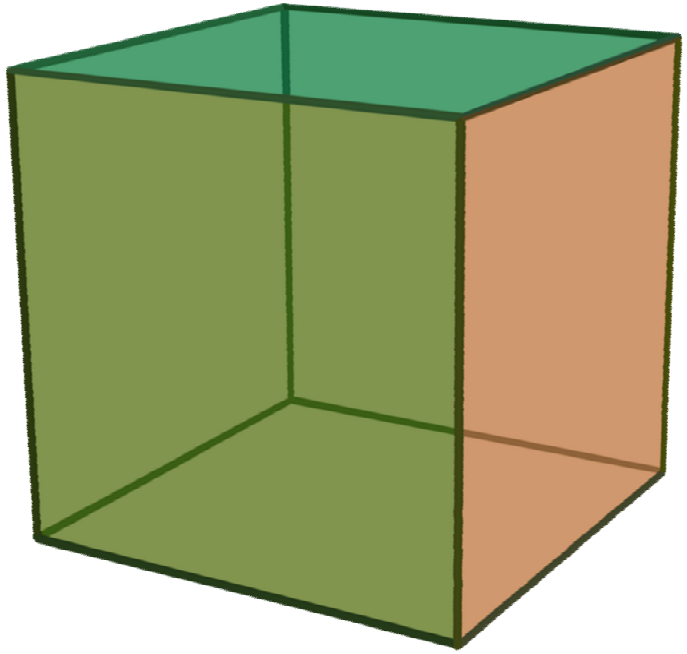
Regular polyhedron

Three squares in each vertex:



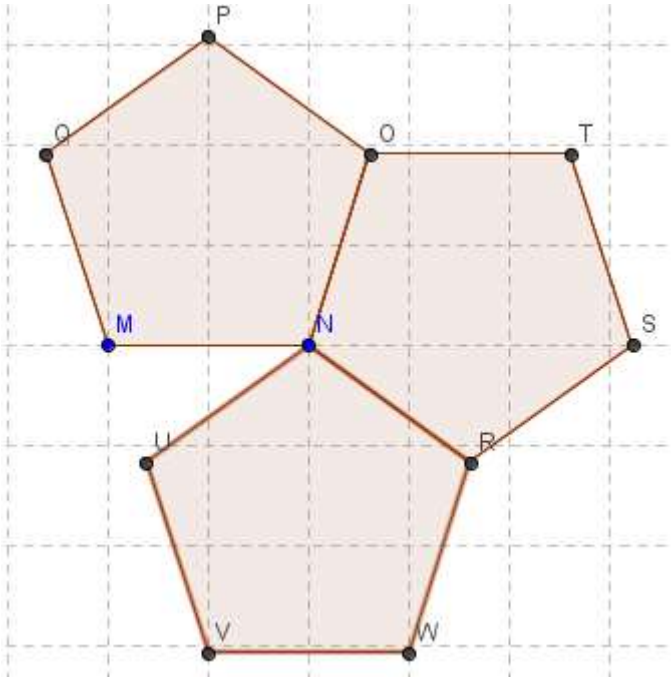


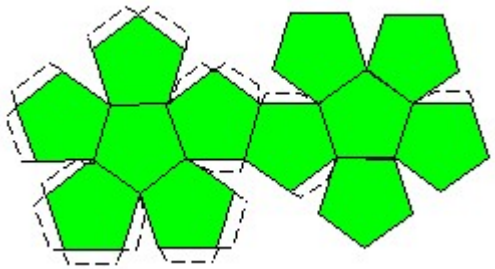
Hexahedron

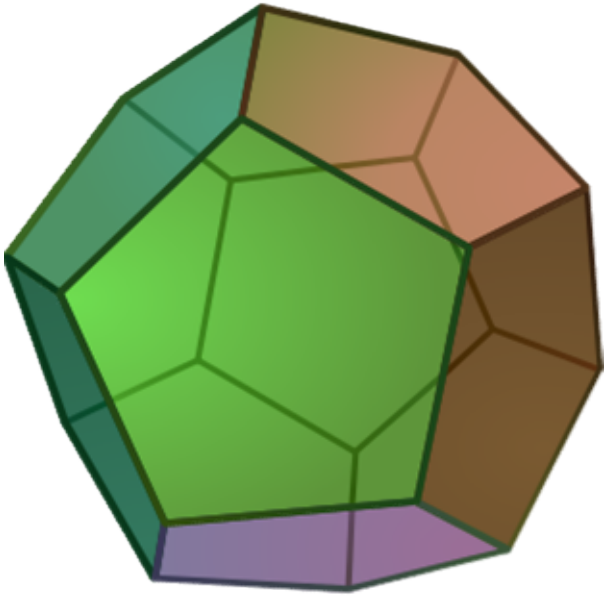


Regular polyhedron

Three pentagon in each vertex:





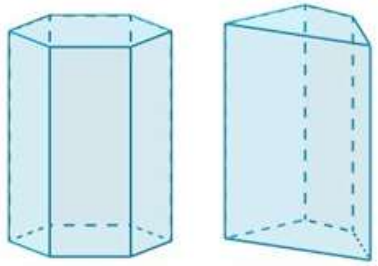


Dodecahedron

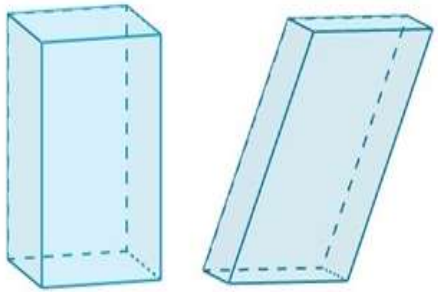
Prisms

- They are polyhedron bounded by two eaqual parallel polygons and the lateral faces are parallelograms.
- The name depends on the bases (the equal polygons).
- The height is the distance betwen the bases.

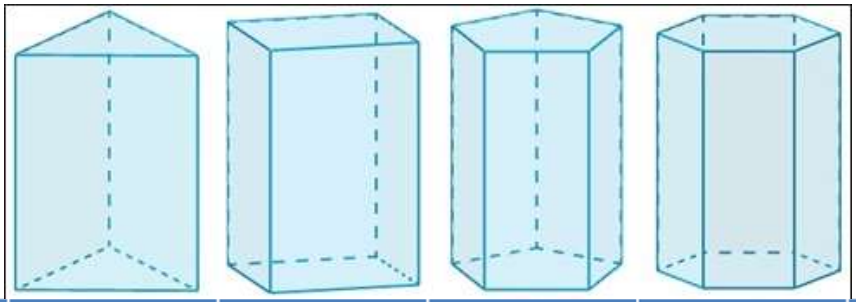
- Regular or Irregular



- Right or General



Prisms

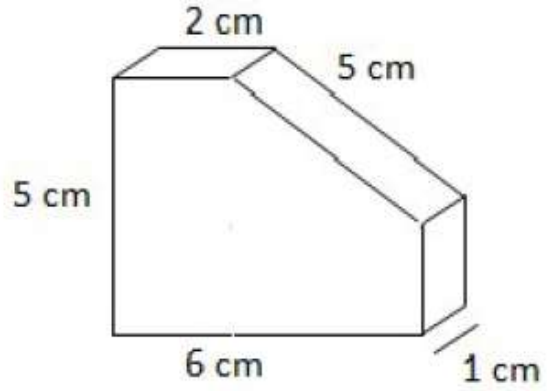


faces	5	6	7	8	...
vertices	6	8	10	12	...
edges	9	12	15	18	...

- The minimum number of faces is 5.
- The number of vertices is always even.
- The number of edges is multiple of 3.

Exercises

1. Compute the area of the surface of the solid of the imagen.



2. Compute the area of the surface of a right regular triangular prism with the edges of the base 5cm and lateral edges of 10cm.

Area and volume

Area:

Lateral area: $p_x h$

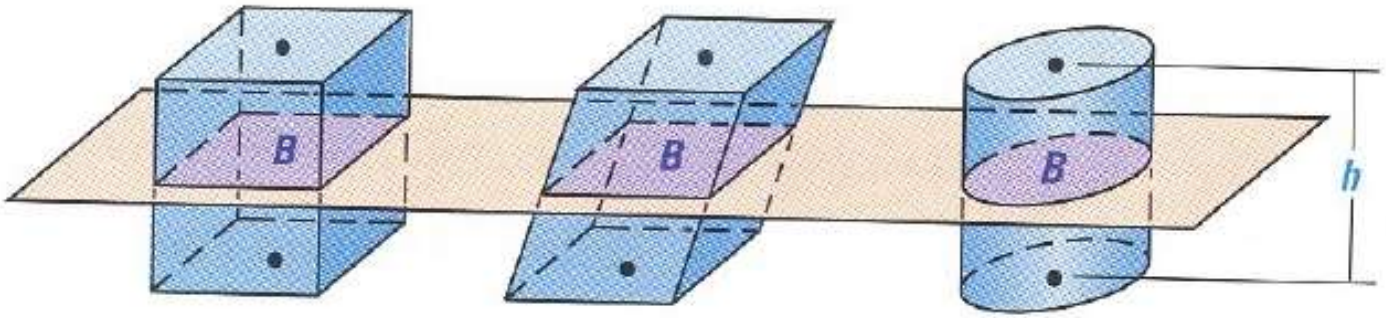
Total area: $p_x h + 2A_b$

Volume:

Volume: $A_b \times h$

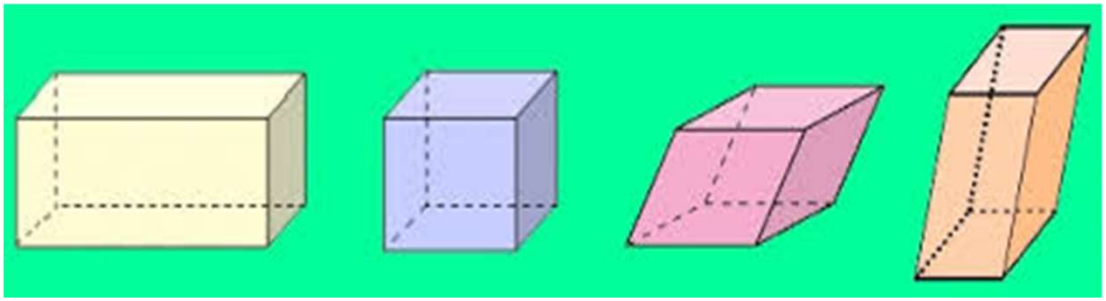
Cavalieri's Principle

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.



Parallelepiped

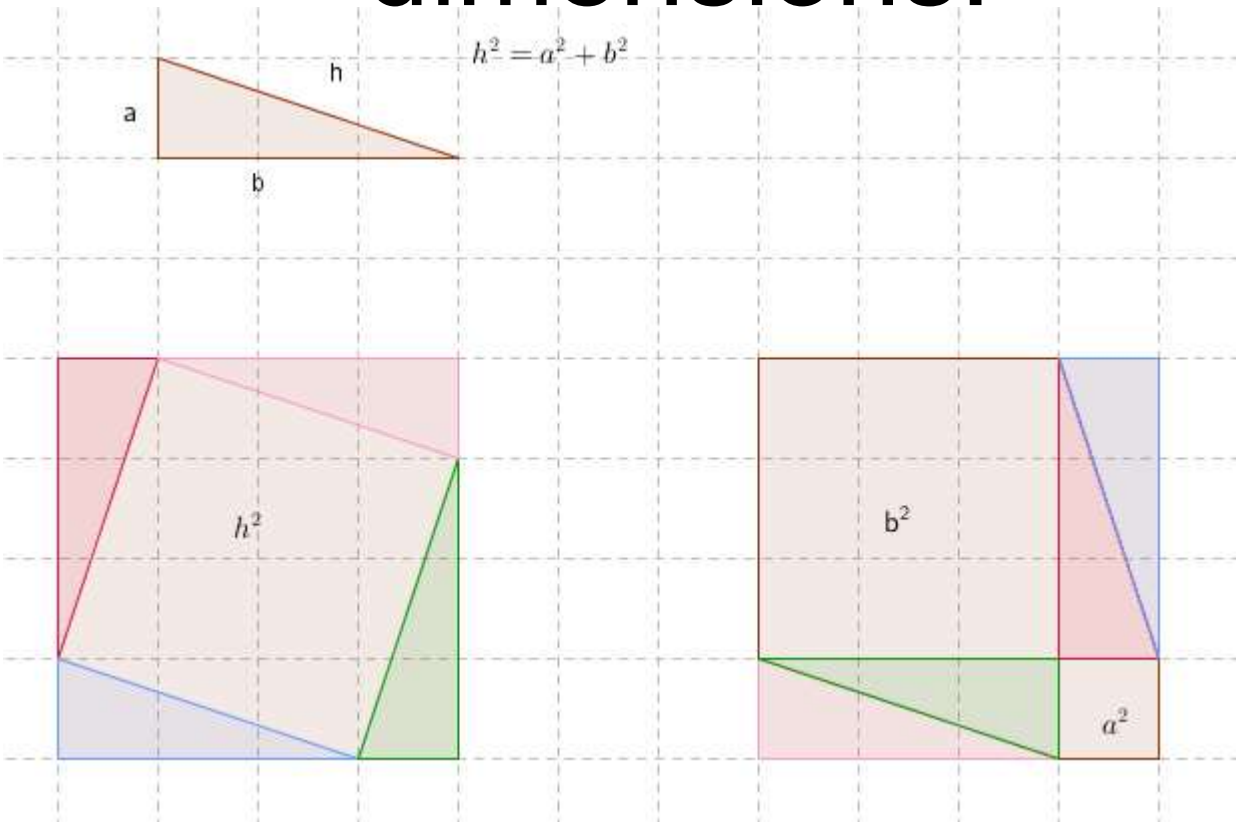
- They are prisms with all the faces parallelograms.



- They verified:
  - The diagonals of a parallelepiped cut in their middle point.
  - In the two first one (bases squares and rectangles), it is verified the Pythagoras theorem in 3-D.

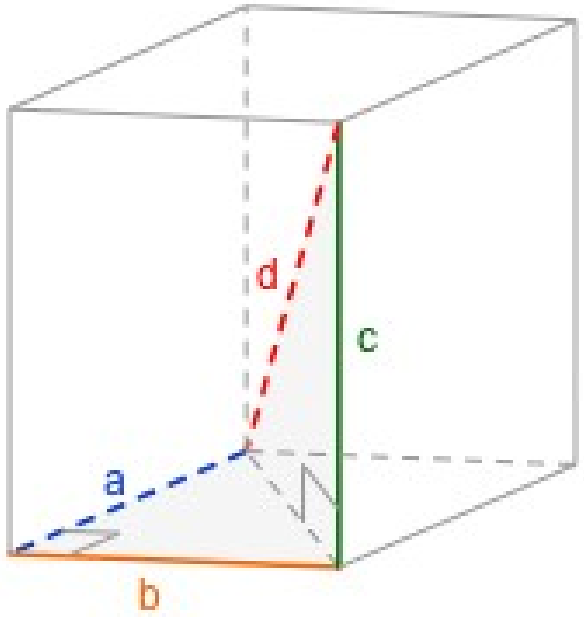
Pythagoras

- Graphical proof in two dimensions:



Pythagoras

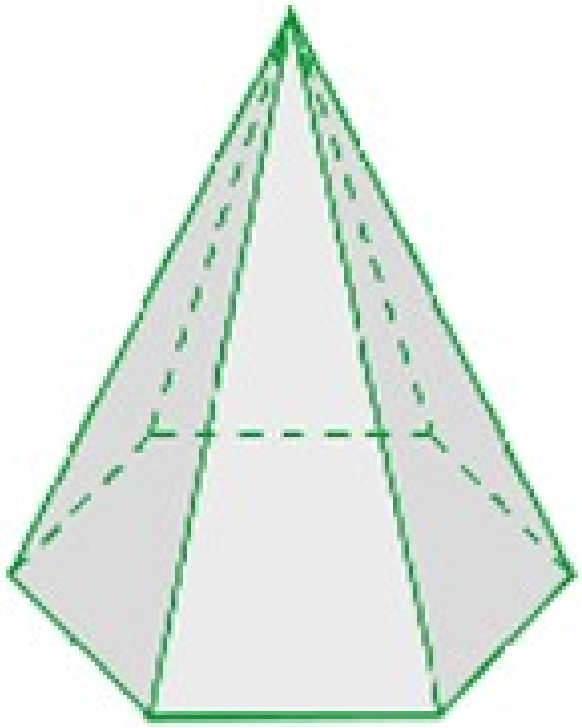
.In 3D.



Note: the diagonal of a cube of edge a is $a\sqrt{3}$

Pyramids

- They are polyhedron bounded by a polygon, called base and lateral faces that are triangles with a common vertex.
- The height or the pyramid is the distance between the base an the vertex.
- The name depends on the base (the equal polygons). If the base is a regular polygon, the pyramid is a regular one and the lateral faces are isosceles triangles.
- The apothem of a regular pyramid is the altitude of the triangles that are its lateral faces.



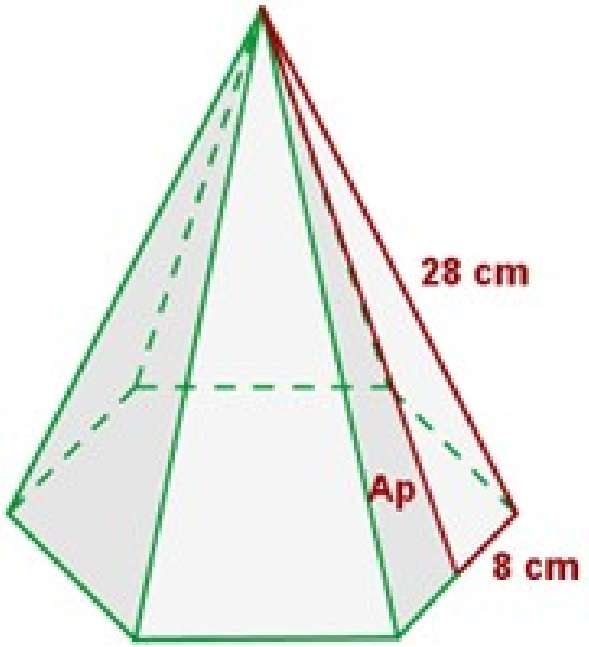
Pyramids

faces	4	5	6	7	...
vertices	5	8	6	7	...
edges	6	8	10	12	...

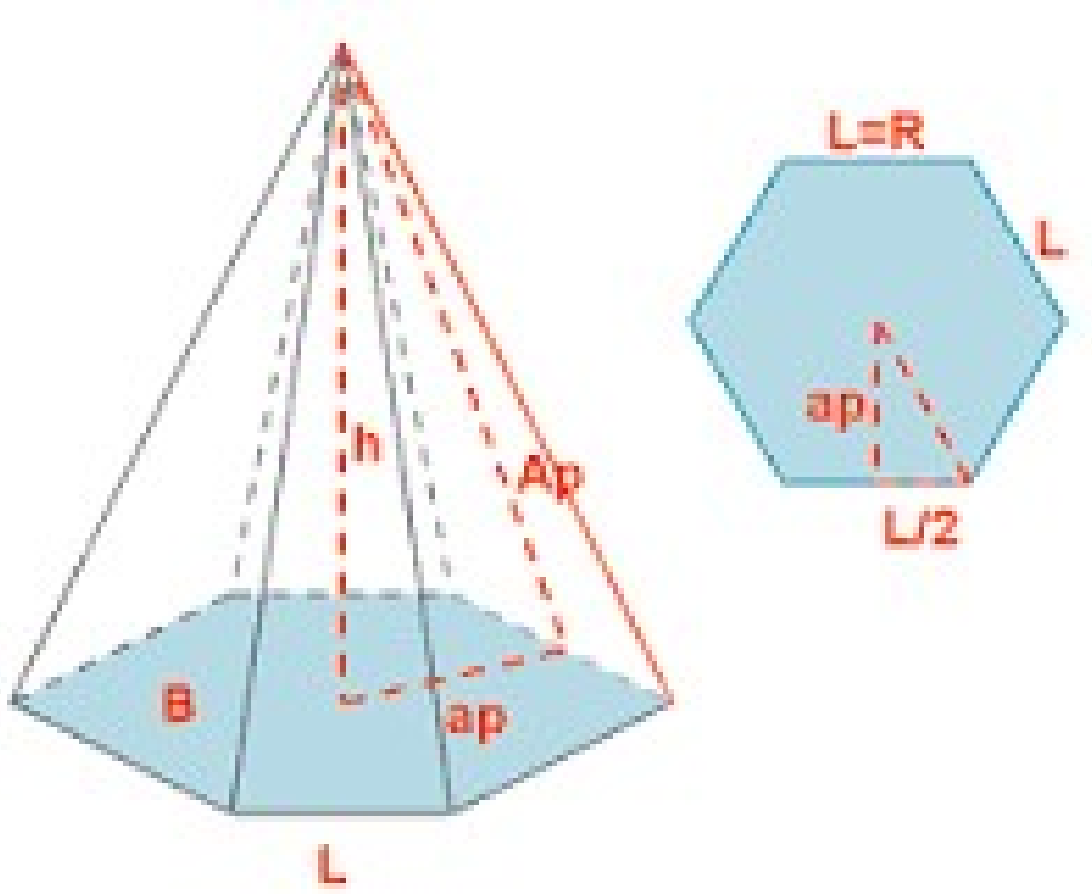
The minimum number of faces is 4.
The number of faces is the same as the number of vertices.
The number of edges is always even.

Exercise

- Compute the apothem of this pyramid:



Be careful with apothems!!



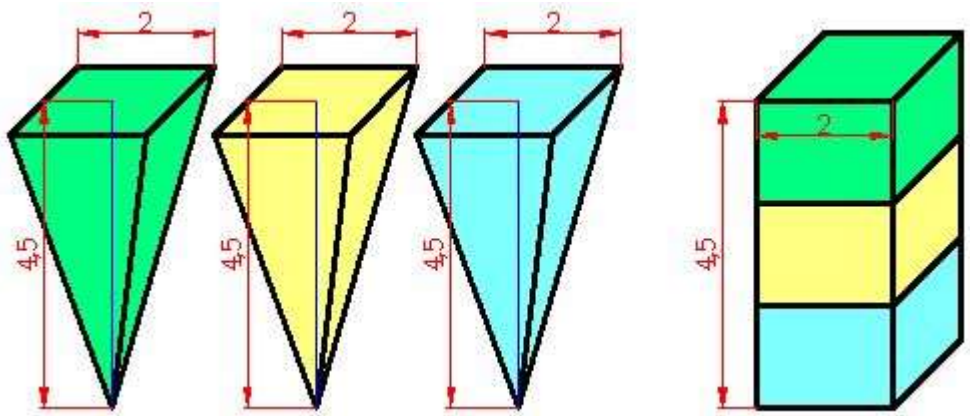
Area and volume

Area:

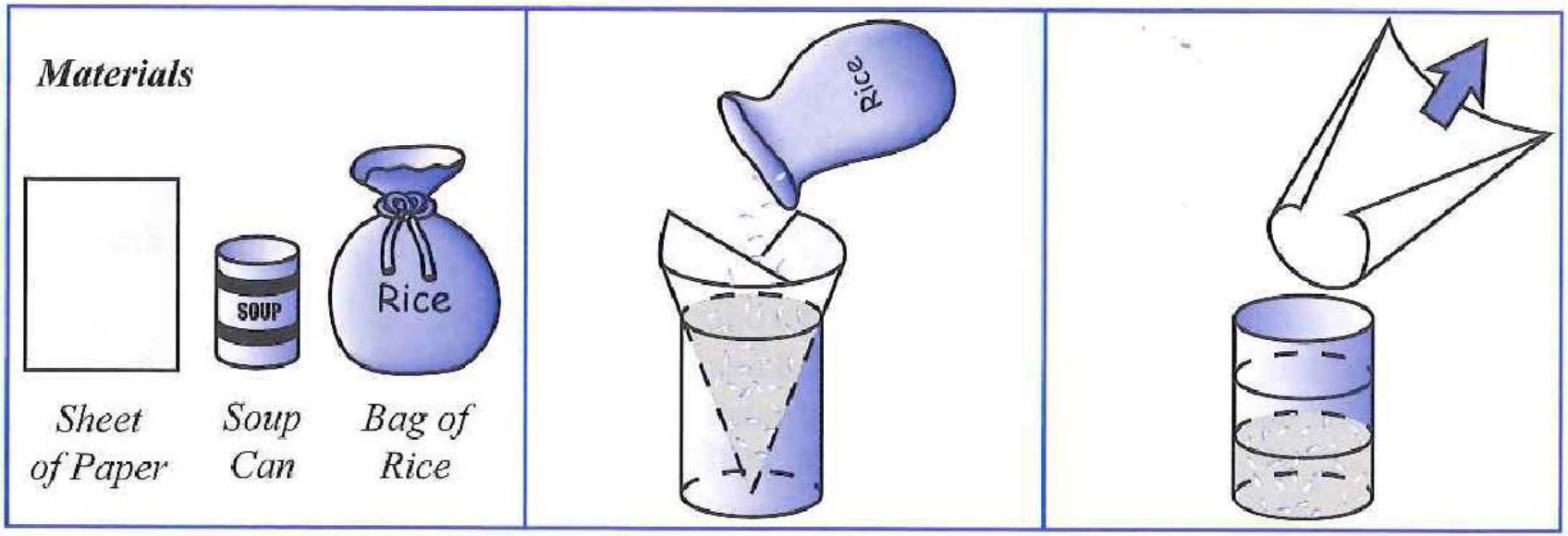
Lateral area: $p \times Ap/2$

Total area: $Ab + p \times ap/2 = p/2\ (Ap + ap)$

Volumen: $Ab \times h/3$



Volumen-Experience



From: Parker T., Baldridge H. (2006) Elementary Geometry for Teachers, Sefton-Ash Publishing, USA.

Exercises

1.a) Which is the minimun number of faces a polyhedron has?

b) A polyhedron has 5 faces and 6 vertices. Draw it.

c) A prism has 12 edges. What is its name?

d) A pryramid has 16 edges. What is its name?

e) Is a prism with 9 vertices possible ? Is a pyramid with 9 edges possible?

f) If the number of sides of the polygon of the base of a prism is multiplied by 2, is the number of faces of the prism multiplied by 2?
And the number of vertices?

g) A polyhedron has as many vertices as edges a tetrahedron has and as many as edges as vertices a hexagonal prism has. How many faces it has?

Exercises

2. Compute the area of a triangle formed with the extrems of three edges with a common vertex of a cube of edge 12 cm. (Extrems that are not the common vertex.)

3. A prism is building, its base is a rhombo with diagonals of 32m and 24m and the height is equal the perimeter of the base. Compute the area of the base and the walls.

Exercises

4. The bases of a right prism are right triangles with area 8 cm2, and the lateral edges are 7 cm. Compute the total area of the prism.

5. Keops pyramid has an square base of side 230m and a height of 137m. It is built using stone blocks of 2500kg each one. Knowing that each m2 of stone is 2800kg. Compute approximately the number of stone blocks using to build the pyramid.

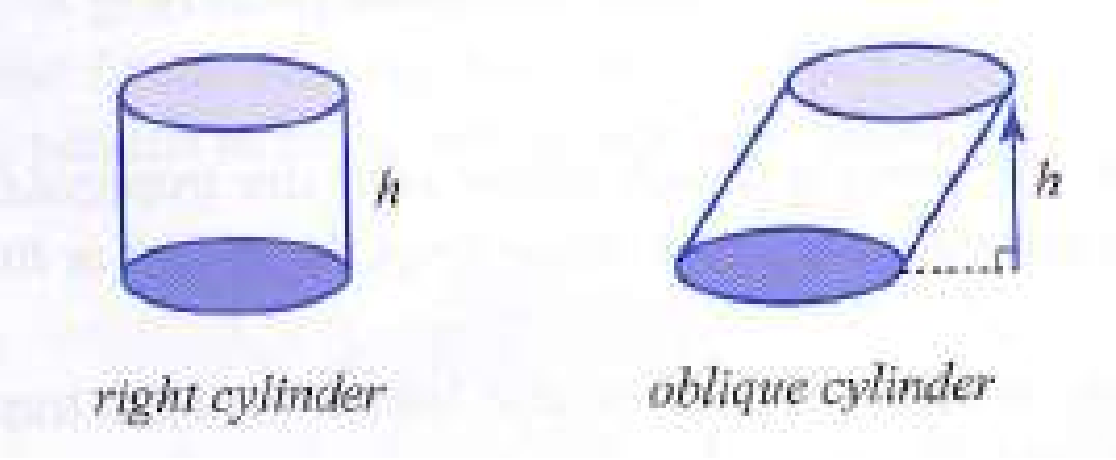
Cylinder, cone and sphere

All of them are obtained by turning a flat figure around an axis called axis of rotation.

Cylinder: It is obtained by turning a rectangle around one of its sides.
Cone: It is obtained by turning a right rectangle around one of its cathetus.
Sphere: It is obtained by turning a semicicumference around a its diameter.

Cylinder

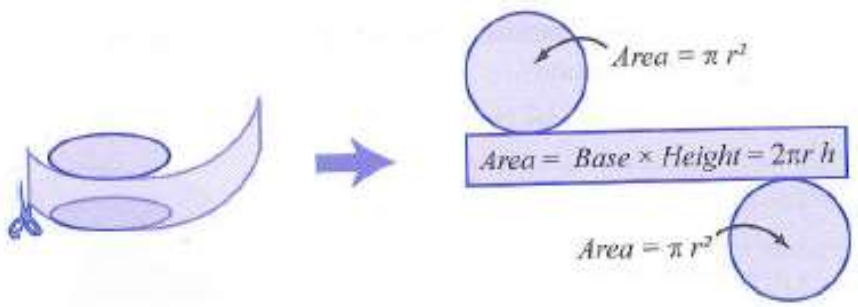
Elements:
-diameter (radio)
-height



Types: right and oblique

Area and Volume

Area:



Lateral area: $2\pi\, r \text{x} h$

Total area: $2\pi\, r \text{x} h + 2\pi\, r^2$

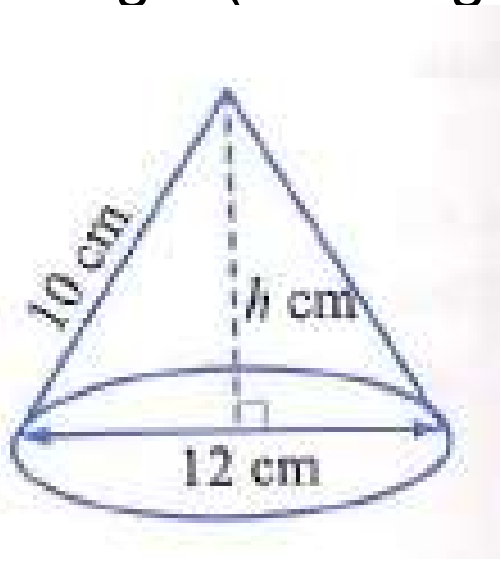
Volume:

Volume $\pi\, r^2 \text{ x } h$

Cone

- Elements:

-diameter (radio),
-height and
-slant height (in the figure 10 cm).



Types: right or oblique

Area and Volume

Area:

Lateral are $\pi\ r \times\ g$

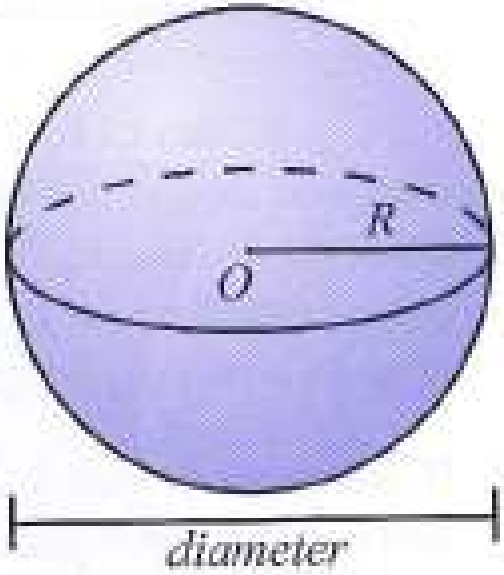
Total area: $\pi\ r \times\ g_{+} \pi\ r^{2}$

Volume:

Volume $\pi\ r^{2}$ x h/3

Sphere

Elements:
-diameter (radio)



Area and Volume

Area:

Total area: $4\pi\ r^2$

Volume:

Volume: $\frac{4}{3}\pi\ r^3$

Exercise

A cylinder with a base radius of 5 cm contains water with a depth of 9 cm. When a stone is immersed in the water, the depth of the water increases to 10.5 cm deep, completely covering the stone. What is the volume of the stone? Write your answer in terms of π.

From: Parker T., Baldridge H. (2006) Elementary Geometry for Teachers, Sefton-Ash Publishing, USA.