

Mathematics and Associated Teaching Methods II

GEOMETRY BASIC CONCEPTS

Degree in Primary Education Teaching

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ORIGIN OF GEOMETRY

- **Geometry**, measure of lands.
- 3000-2300 bC Sumerian civilization
 - Multiplication and Division
 - Geometrical problems
- 2000-1600bC Ancient Babylonia
 - Estimation of $\pi \rightarrow 3$ (similar to Bible or Simpsons)
 - Use Pythagoras Theorem
 - Define units to measure distance and time
- **1300bC Herodoto (Ancient Egypt)**
 - measure areas

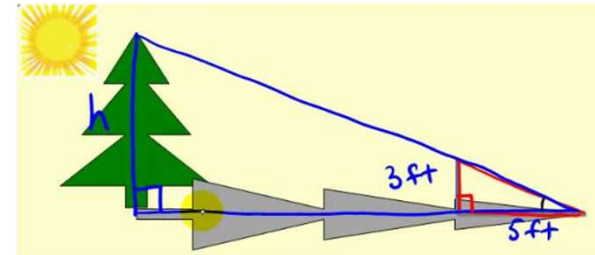


ORIGIN OF GEOMETRY

Ancient Greek (900bC-600aC)

Thales (624bC - 546bC):

- Proportionality



Pythagoras (596bC - 475bC):

- Theorem
- Irrational numbers



Euclid (324bC - 265bC):

- Deductive geometry. "The elements"

ORIGIN OF GEOMETRY

The elements

Five axioms:

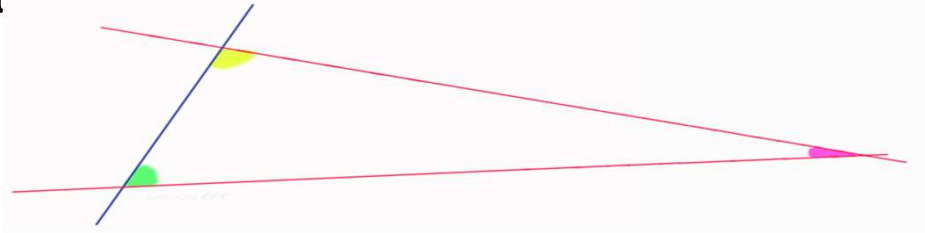
A1."To draw a straight line from any point to any point."

A2."To produce [extend] a finite straight line [segment] continuously in a straight line."

A3."To describe a circle with any centre and distance [radius]."

A4."That all right angles are equal to one another."

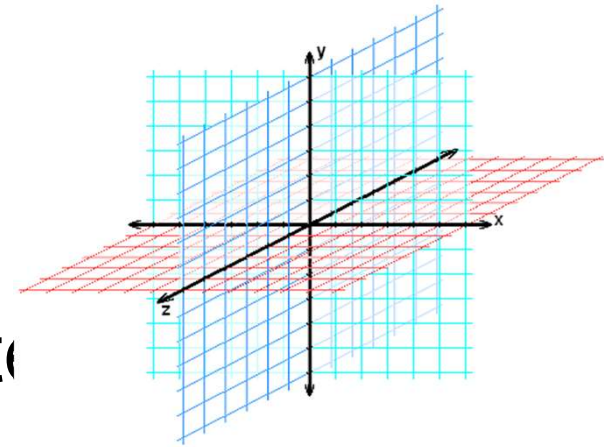
A5. The parallel postulate: "That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles."



ORIGIN OF GEOMETRY

1500 aC Descartes

- Cartesian coordinate system
- Analytical geometry .
- For instance, $(x-0_1)^2 + (y-0_2)^2 = r^2$.



Basic Definitions

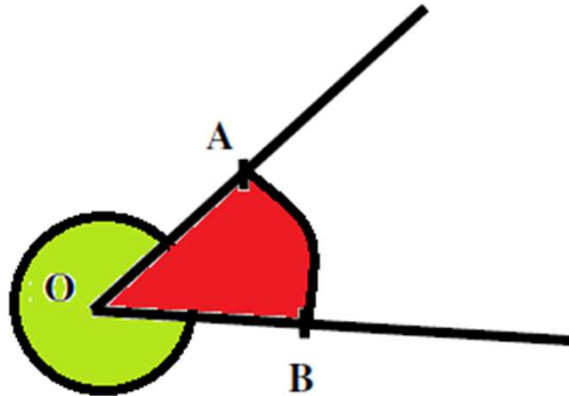
- **Plane.** Flat surface that extends without edges.
- **Line.** a long mark of very slight thickness without end in both directions. (**straight line:** with no change of direction)
- **Point,** what can not be divided in parts (Euclid)

Basic Definitions

- Semiplane
- Ray
- Segment
 - Successive segments
 - Consecutive segments: successive segment in the same straight line.

Basic Definitions- angles

- Two rays with the same origin that separate the plane in two infinite regions.



Basic Definitions- angles

- Angles depending on their location:
 - Adjacent angle
 - Adjacent angles on a straight line
 - Opposite (or vertical) angles
- Angles depending on their opening:
 - Straight angle
 - Right angle
 - Acute angle
 - Obtuse angle
 - Null

Basic Definitions- angles

Measuring angles

Sexagesimal System

Total=360°, 1°=60', 1' = 60"

Centesimal System

Total=400°, 1°=100', 1'=100"

Mixed System

It is sexagesimal with decimals. For instance, 32,5° in stead of 32° 30'

Operations with angles

- Sum: $42^{\circ} 58' 35'' + 13^{\circ} 17' 22'' =$
- Subtraction: $18^{\circ} 20' 38'' - 13^{\circ} 43' 50'' =$
- Multiplication: $15^{\circ} 43' 52'' \times 6 =$
- Division: $43^{\circ} 18' 10'' / 5$

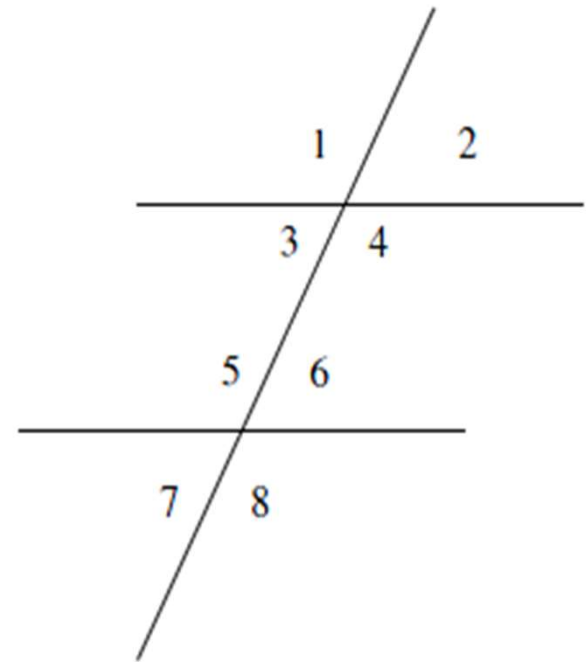
More definitions

- Complementary angles
- Supluntary angles
- Parallel
- Perpendicular
- Bisector of an angle
- Perpendicular bisector of a segment

More definitions

Two parallels and a transversal line

- Interior angles: 3,4,5,6,
- Exterior angles: 1,2,7,8
- Corresponding angles - angles that are in the same position on each line. (1,5), (2,6), (3,7), and (4,8).
- Alternate interior angles - angles on the opposite sides of the transversal and on the interior of the parallel lines with different vertex. (4,5), and (3,6).
- Alternate exterior angles - angles on opposite sides of the transversal and on the exterior of the parallel lines with different vertex. (7, 2), and (1,8) .



Properties

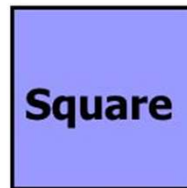
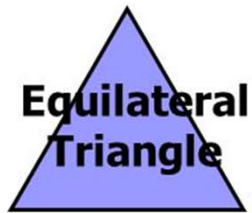
- Alternate interior angles are equal
 - Alternate exterior angles are equal
 - Corresponding angles are equal.
-
- Exercise: Find all the angles formed when crossing two parallels by a transversal
Knowing angle 2=30°.

POLYGONS

- Polygonal path: A set of successive segments. It could be:
 - opened or closed
 - convex or concave
- Polygon: A part of a plane boudary by a closed poligonal path (it is called polygonal region). It could be:
 - Convex or concave.

POLYGONS

NAMING REGULAR POLYGONS



Some definitions

- Perimeter: The sum of the lengths of the sides of the polygon.
- Diagonal: are line segments from one corner to another (but not the sides).
- How to calculate the number of diagonals of a n -gon.

Some definitions

- Interior angles: angles in the polygonal region with a vertex on the vertex of the polygon.
- The sum of the interior angles of a n -gon.
- Exterior angles: The Exterior Angle is the angle between any side of a shape, and a the prolongation of the line from the next side.
- The sum of exterior angles of a n -gon.

Triangles

Definition: A polygon with 3 angles.

- Triangle inequality: $AC \leq AB + BC$
- Classification of triangles:
 - By sides
 - By angles

Triangles

BY SIDE

BY ANGLE

Equilateral Triangle



-has three equal sides

Acute Triangle



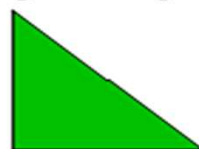
-three angles < 90 degrees

Isosceles Triangle



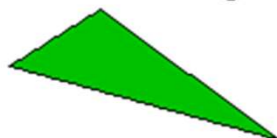
-has two equal sides

Right Triangle



-has one right angle

Scalene Triangle



-has no equal sides

Obtuse Triangle



-has one angle > 90 degrees

Triangles

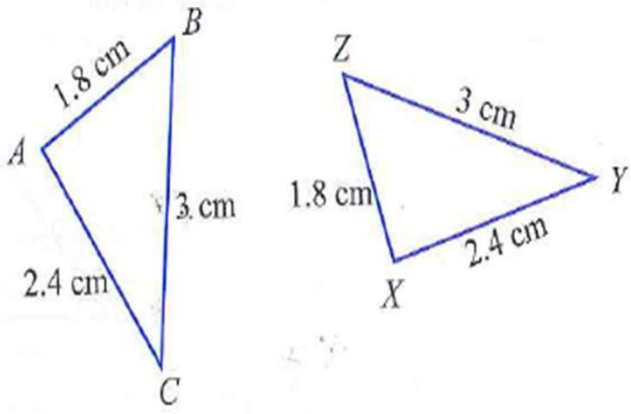
- Two triangles are congruent if:
 - All pairs of corresponding sides are equal

EXERCISE 2.3. These triangles are congruent. Fill in the blanks by matching side lengths and estimating angle measures.

$\overline{AB}$ corresponds to $\underline{\hspace{1cm}}$
 $\overline{BC}$ corresponds to $\underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}}$ corresponds to $\overline{XY}$

$\angle A$ corresponds to $\angle \underline{\hspace{1cm}}$
 $\angle B$ corresponds to $\angle \underline{\hspace{1cm}}$
 $\angle \underline{\hspace{1cm}}$ corresponds to $\angle Y$.

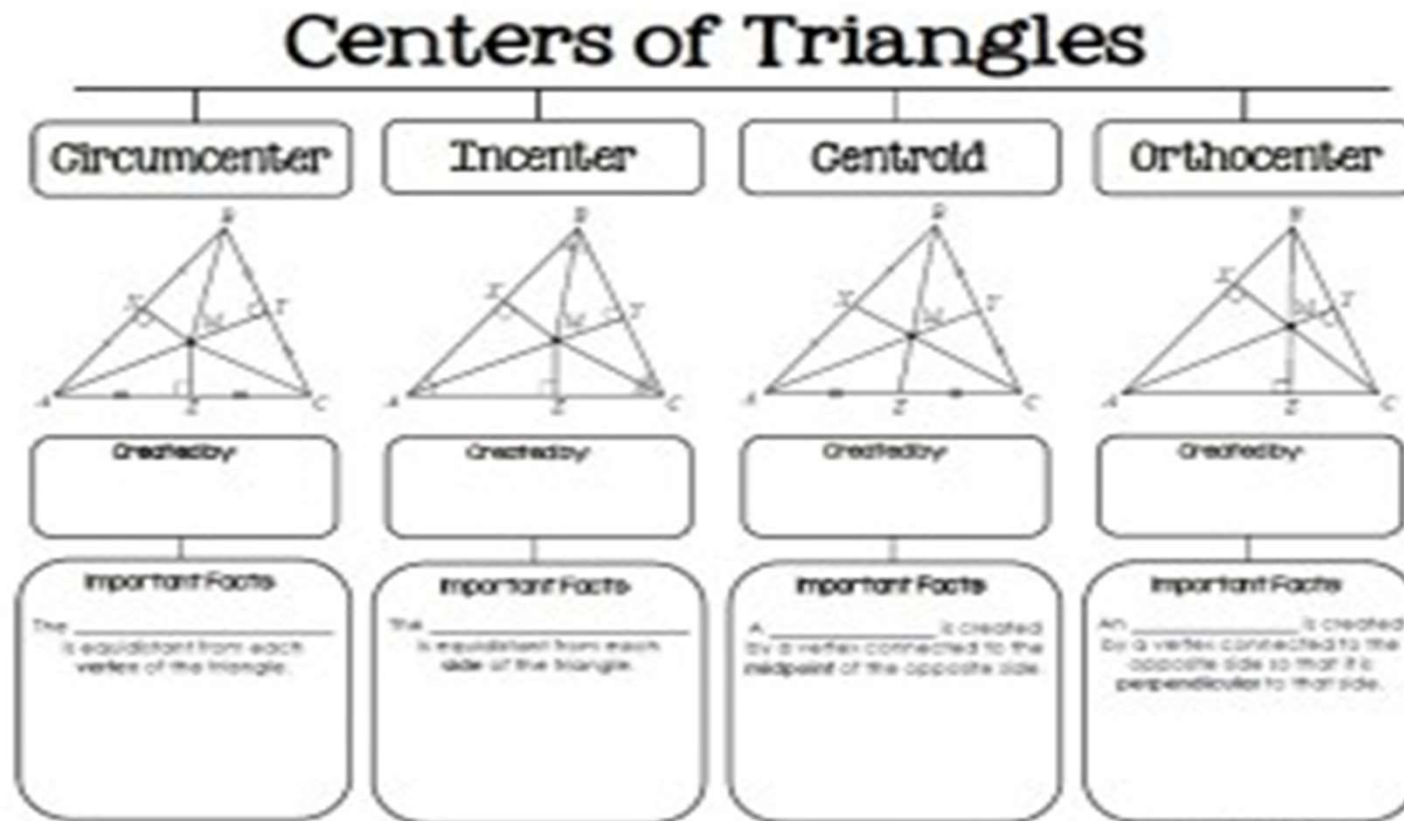
$\therefore \triangle ABC \cong \triangle \underline{\hspace{1cm}}$.



Equality of triangles

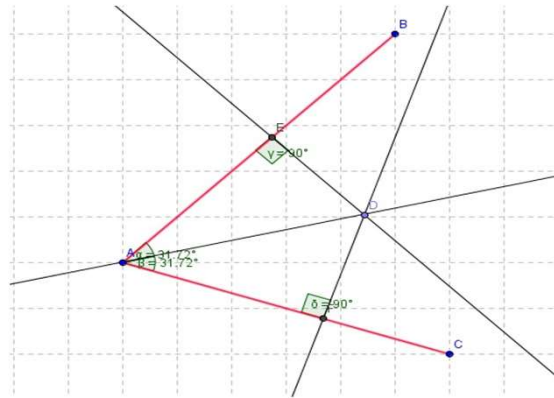
- Two triangles are CONGURENT:
 - If they have two angels equal and the side between them.
 - If they have two sides and the angle between them equals.
 - If they have the three sides equals.
 - If they have two sides equal and the angle just opposite the the longest side.

Main elements in a triangle



Bisector - Incenter

- The three bisectors of a triangle meet at a point called incenter:
- It is the center of the inscribed circle.
- Any point on the bisector of an angle is at the same distance to the sides of the angle.



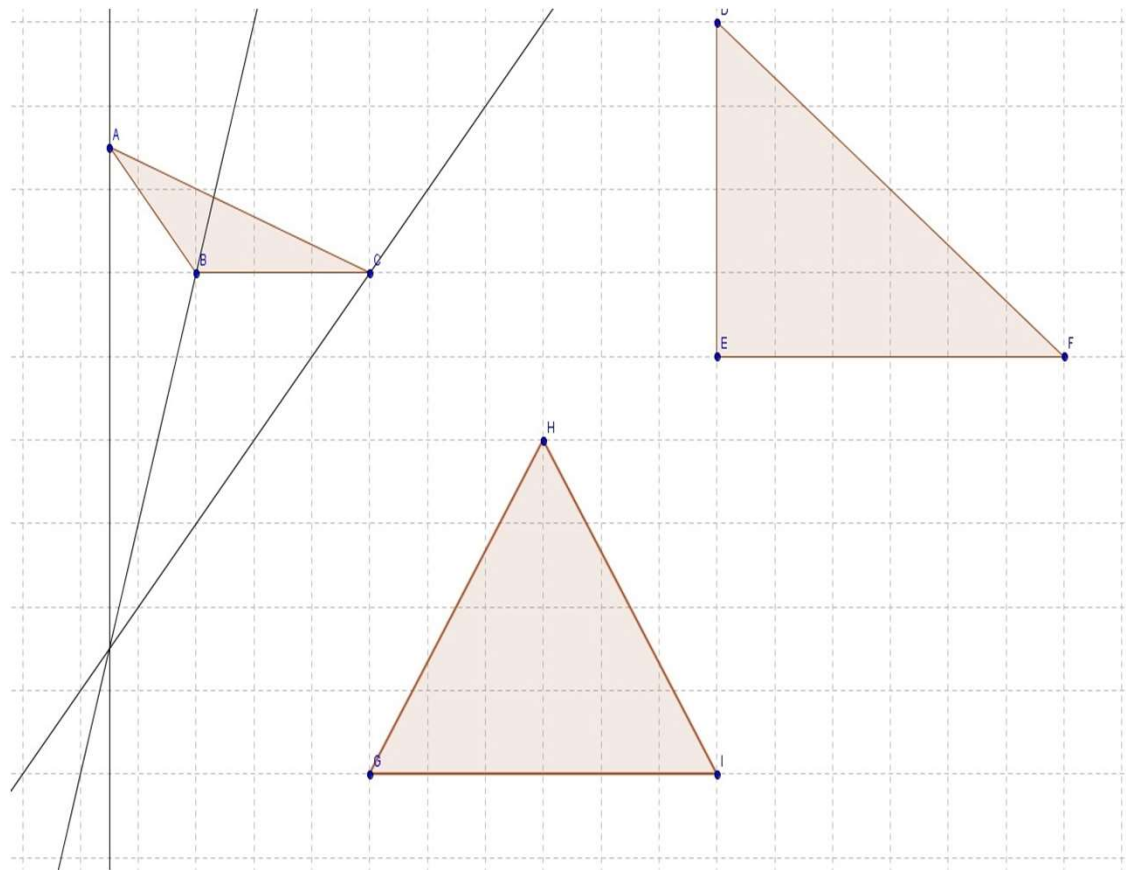
Perpendicular bisector of all the sides- Circuncenter

- The three perpendicular bisector of the three sides of a triangle cut in a point called circuncenter:
- It is the center of the circumscribed circle.
- Any point in the perpendicular bisector of an segment is at the same distance to the extremes of the segment.

Heights of the triangle - Orthocenter

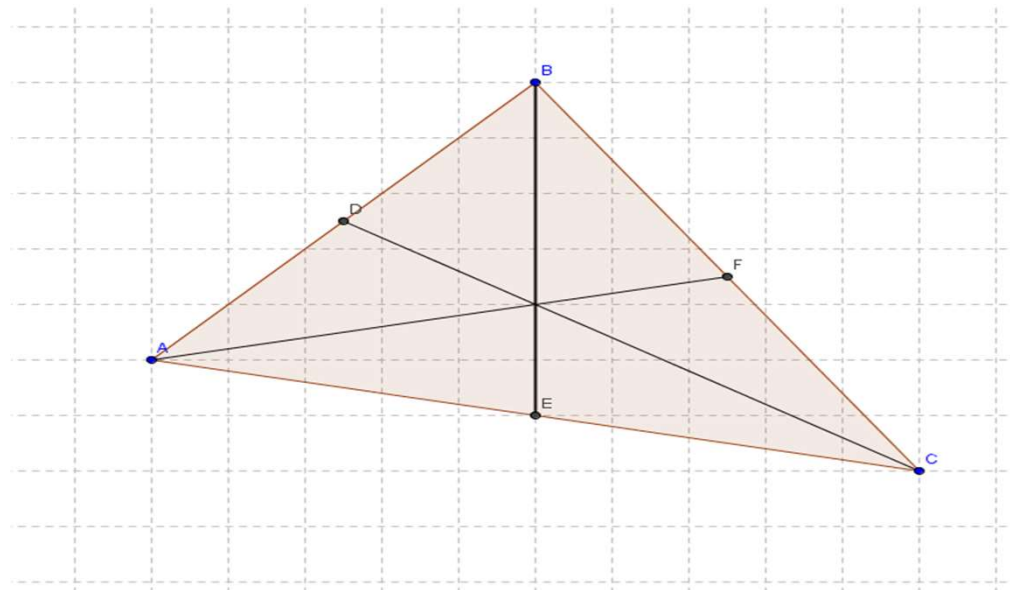
- Altitude $\rightarrow$ corresponding to a base.
- It is a segment perpendicular to the base from any point on the opposite vertex.
- The three altitudes of a segment cut in a point called orthocenter.

Heights of the triangle - Orthocenter



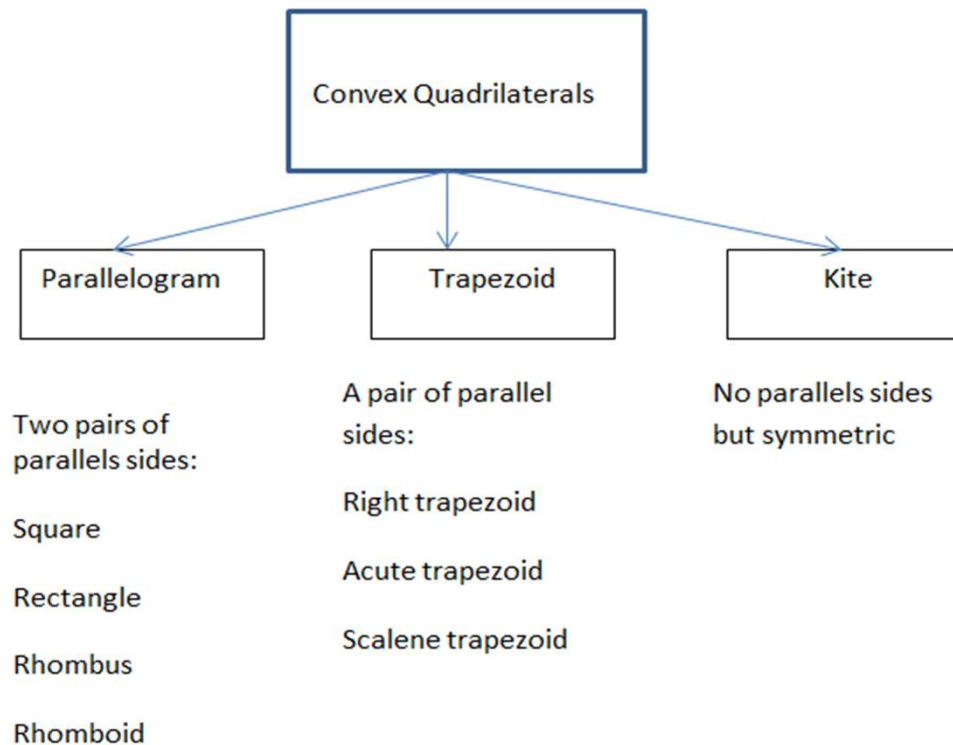
Medians-Centroid

- Median of a triangle: It is a segment from a vertex to the middle point of the opposite side.
- The three medians cut in a point called centroid.
- The centroid divides de median in two parts: $\frac{1}{3}$ and $\frac{2}{3}$ of the total lenght.



Quadrilaterals

- A polygon of 4 sides and four vertex. (convex or concave)
- Convex quadrilateral classification.



Properties

1. Parallel segments delimited by parallel lines are equal.

Consequence:

- 1.1. Any parallelogram has opposite sides equal.
- 1.2. Any parallelogram has opposite angles equal.
- 1.3. Any diagonal of the parallelogram divides it in two equal triangles.

Properties

2. The point in which two diagonals of a parallelogram touch divides each diagonal in two equal parts.

Go deeper:

3. An **square** has two equal perpendicular diagonals that are bisector of the angles.

4. A **rectangle** has equal diagonals that are not perpendicular.

5. A **rhombus** has two different diagonals that are perpendicular and bisector of the angles

6. A **rhomboid** has two different diagonals that are not equal and not perpendicular.

Properties

7. In a triangle if a line is drawn from the middle point of a side parallel to other side. It is obtained a point in the third side that is just in the middle and the segment drawn inside the triangle is the middle length of the side that is parallel.
8. The segment delimited by the middle point of two sides of a triangle is parallel to the third side and the length is the middle of the third side.
9. If the three middle points of the sides of a triangle are joined through segment, the triangle is decomposed in 4 equal triangles.

Properties

10. In a trapezoid, the segment with endpoints the middle point of the non parallel sides is parallel to the bases and the length is the middle of the sum of the length of the bases.