EXERCISES: SET THEORY

1. Indicate if the following sets are: empty sets, singleton sets, finite sets, infinite sets.
   - A={natural numbers lower than 20}
   - B={natural numbers greater than 20}
   - C={natural numbers lower than 20 and greater than 20 simultaneously}
   - D={days of the week containing the letter g}
   - E={n ∈N | n odd, n > 4}, where N={natural numbers}
   - F={n ∈N | n even, n < 4}
   - G={x | x is a vowel}
   - H={n ∈N | 0 < n < 1}
   - I={n ∈ N | n prime, 1 < n < 10}
   - J={x ∈T | x is scalene and isosceles}, where T={plane triangles}

2. Indicate at least one element belonging to each one of the sets provided in the Exercise 1, and another element which does not belong to it. Employ the usual set notation.
   For instance: 1 ∈A , 100 ∉A, 19 ∈ A, monday ∉ A

3. If it is possible define the sets in Exercise 1 by using the extensional definition.

4. Consider the universal set U = {a, o u, c, d, l, m, s}. The following subsets are considered:
   A = {c, a, s, o} and B = {m, a, l, o}.
   - Calculate $A \cup B, A \cap B, A - B, B - A, \bar{A}, \bar{B}, A \cup \bar{B}, \bar{A} \cap \bar{B}, \overline{A \cup B}, \bar{A} - \bar{B}$
   - Represent the sets A, B and U in a Venn diagram.

5. Given the universal set U = {a, o, u, c, d, l, m, it is known that A ∪B= {a, l, o, m, c}, A – B = {m} and A ∩B = {a, l, o}.
   - Calculate A and B.
   - Represent the sets A, B and U in a Venn diagram.

6. Given the universal set of the first ten alphabet letters, the subsets A={a,b,c,d,e}, B={vowels} and C={vowels of the word "archeologist"} are considered.
   - Represent all the given sets in the same Venn diagram.
   - Calculate $\bar{B} \cap A$, $A \cap (B \cap C)$ and $C - B$.

7. Draw in a Venn diagram three non-empty sets A, B, C satisfying the following conditions (there may be more than one possibility):
   a) A ⊂ B, C ⊂ B, A ⋂ C = ∅
   b) A ⊂C, B ⋂C = ∅
   c) A ⊂ B, C ⊄ B, A ⋂ C ≠ ∅
   d) A ⊂ (B ⋂ C), B ⊂ C

8. In the set of all people we define the sets H = {men}, E = {European people} and D = {sporty people}. Represent graphically the sets $\bar{E} \cup \bar{D}$ and $(E \cap \bar{H}) \cap D$. Point out which are the characteristics of the elements belonging to each one of the last sets.

9. In the set of people in a swimming pool we consider the sets A = {people who know how to swim}, B = {people wearing black swimsuit}, C = {people wearing a cap}.
   By using the usual set notation express each one of the following situations, representing them in Venn diagrams (and exploring all the possibilities):
   a) All those who wear a black swimsuit know how to swim, but not all those who wear a cap know how to swim.
   b) All those who wear a cap also wear a black swimsuit, and all those who know how to swim wear a black swimsuit.
   c) All those who know how to swim wear a cap and they also wear a black swimsuit. However, not all those who wear a black swimsuit also wear a cap.

10. Consider the universal set U={students of the faculty}, and let A={first year students}, B={second year students}, C={Primary Education students}, D={Early Childhood Education students}, E={Music students}, F={students who went to the concert yesterday}, G={students who went to bed late yesterday}.
    Express each one of the following situations by employing the usual set notation:
    a) All the first year students of the Primary Education degree study Music.
    b) The Music students who went to the concert yesterday also went to bed late.
    c) No Early Childhood Education student went to the concert yesterday.
    d) The concert was made only for first and second year students who study Music.
    e) All second year students who do not study the Early Childhood Education degree went to the concert and went to bed late.

11. In a group of 100 people, there are 70 of them who read *La Nueva España,* 60 who read *El Comercio* and 20 who do not read any of the two newspapers. How many people read both newspapers?

12. In a classroom of 44 students, there are 20 of them who are taking a Language exam and 18 who are taking a Maths exam. If there are 10 of them who are taking a Language exam but not a Maths one:
    a) How many of them do not take any exam?
    b) How many are taking any of the two exams?
    c) How many are taking both exams?

13. The cards of 40 users of a library are checked. From them we know that 20 users read adventure books and 18 users read mystery books. In addition, if it is also known that 15 users read mystery but not adventure books, then calculate:
    a) How many of them did not read books of any of these two categories?
    b) How many read adventure but not mystery books?

14. From 500 people consulted, there are 325 men, 168 single people and 93 single men. How many people are women who are not single?

15. In a languages academy it is taught, among other languages, Chinese and Arabic. There are 50 students and, from them, 23 study Chinese, 9 Arabic and 20 no one of the two languages. How many study both languages? How many study Arabic but not Chinese?
    English is also offered as a third language in the academy. There are no students studying the three languages, but there are 6 of them studying English and Chinese and 5 studying English and Arabic. If all the students in the academy are studying at least one of the three languages, how many people study only one of them?

16. In a classroom of 30 students, 16 of them play football, 14 play basketball, 13 play tennis, 6 play football and basketball, 6 football and tennis, 5 basketball and tennis and, finally, 4 play the three sports. How many of them do not play any of the three sports? How many play tennis and football but not basketball?

17. In a survey about musical styles, we have obtained that the 40% of the respondents like reggaeton, the 75% like rock and the 53% like classical music. In addition, the 35% like reggaeton and rock, the 30% like rock and classical music, and the 5% only like reggaeton and classical music. If the 10% like the three music styles, what is the percentage of people who only like reggaetón? What is the percentage of people that do not like any of the three styles?

18. Some painting materials (pencils and markers) are mixed in a big box. The number of pencils is the same that the number of markers. In addition, 30 pencils as well as 45 markers are blue. 42 pencils are not blue. How many markers are not blue? How many objects are there in the box?

19. Each one of the 100 students in a class do 3 exams. 40 of them pass the first, 39 the second and 48 the third, and 10 students pass the three exams, 21 do not pass any of them, 9 pass the first and the second but not the third and 19 pass the third but not the other two. How many students pass at least two exams? There are some unnecessary data, which are them?

20. In order to know the habits of the asturian people about social networks we have asked 150 people if they have an active account in some of the most famous social networks: Facebook, Tuenti or Twitter. Taking into account the positive answers, we obtain that 25 people have only Facebook account, 30 have only Tuenti account, and 20 have only Twitter account. 15 of them have Facebook and Tuenti accounts, but not a Twitter one. Only 5 people have an account in the three social networks. After the experiment, we obtain 65 Facebook users, 55 Tuenti users and 50 Twitter users. How many respondants do not have an account in any of the three social networks? How many users have exactly two social networks?

21. In a classroom of 84 students, 38 are boys. From them, 23 use glasses and 27 have blonde hair. There are 51 students in the class using glasses and 63 who have blonde hair. In addition, there are 12 blond boys with glasses. How many blond boys do not use glasses? How many girls use glasses?

22. In a group of friends, 37 of them smoke, 25 drink and 17 chat; 8 smoke and drink, 5 smoke and chat, and 3 drink and chat. There is only one person that do the three things whereas there are 16 that do not do any of the three. How many Friends are they?

23. In a Faculty of Education the students that fail in Mathematics, Physical Education and Plastic Arts must repeat the second school year. The results obtained the last year were the following: the 5% of the students passed the three subjects, the 20% passed Mathematics and Plastic Arts; the 15% passed Mathematics and Physical Education; the 25% passed Physical Education and Plastic Arts; the 35% passed Mathematics; the 50% passed Physical Education, and the 45% passed Plastic Arts. What is the proportion of students that repeated the second school year?

24. In my neighborhood there are 70 families and some of them have pets. 30 families have a dog, 18 have a cat and 13 have a bird. In addition, 3 families have a dog and a cat, 8 have a dog and a bird, but no one of them dare to have a cat living together with a bird.
    a) How many families have the three pets?
    b) How many cats are the only pet in the house?
    c) How many families do not have pets at home?
    d) How many pets are there in total in the neighborhood?

25. From 24 students of a class of the 6th grade of Primary Education, it is known that 3 of each 4 students have a mobile, 2 of each 3 students have a computer and 1 of each 12 do not have neither mobile nor computer. How many students have mobile and computer?

26. In a 3th grade course there are 30 students. We know that a 60% live in Oviedo and a 70% goes to school by feet. In addition, the 20% does not live in Oviedo and employs a means of transport to go to school. How many students live in Oviedo but use a means of transport to go to school? How many students do neither live in Oviedo nor go by feet to school?

27. We have done a survey to a group of sporty people and we have obtained that 115 play basketball, 35 play basketball and chess, 90 play only chess, and 105 do not play chess. How many sporty people have answered the survey? How many of them play only basketball?