Mathematics and Associated Teaching Methods I

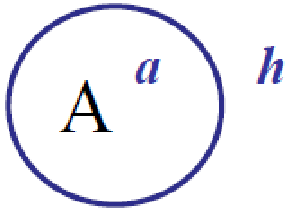
SET THEORY

BASIC CONCEPTS

Degree in Primary Education Teaching

FIRST DEFINITIONS

- SET: collection of distinct objects, considered as an object in its own right.
- ELEMENTS (OR MEMBERS) OF A SET: each one of the objects that make up a set.
- Notation: -Set: A; -Elements of A: *a, b, c, d*,...
- Mathematical representation: A={*a,b,c,d*}
- Graphical representation: Venn diagrams



Membership Relation

- Given a set A, the membership relation to A for any object is defined as follows:
  - If a is an element of A: "a belongs to A"

$$a \in A$$

  - If a is not an element of A: "a does not belong to A"

$$a \notin A$$

Types of sets

- A set is well defined whenever given any object it is possible to determine if it belongs or not to that set.

- According to the number of elements of the set:
  - Finite set.
  - Infinite set.

Ways of describing a set

- Extensional definition: the list of all the elements of the set is enclosed in curly brackets.

$$A = \{a, b, c, d\}$$

- Intensional definition: it is established a rule or semantic description which is satisfied by all the elements.

$$A = \{x \mid x \text{ satisfies the property } p\}$$

| means "such that"

Examples

- Set of natural numbers lower than 6:
  - Extensional definition: $A = \{1,2,3,4,5\}$
  - Intensional definition: $A = \{n \in N \mid n < 6\}$

- Set of all the Andalucian provinces.
- Set of the natural numbers between 2 and 10.
- Set of natural numbers lower than 40.
- Set of natural numbers greater than 6.

The infinite sets can only be defined by means of an intensional definition

Special types of sets

- <u>Singleton Set:</u> the one having only one element

$$A = \{2\} = \{n \in N \mid n \text{ even lower than } 4\}$$

- <u>Empty Set or Null Set:</u> the one having no elements.

$$\{n \in N \mid n \text{ even lower than } 2\} = \phi$$

- <u>Universal Set:</u> the one having all the elements of the reference framework.

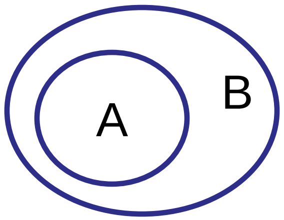
$$U = \{\text{natural numbers}\}$$

Subsets

- Given two sets A and B:
  - "A is a subset of B"
    "A is contained in B"
    "A is included in B"

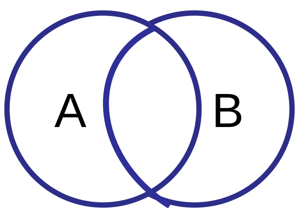
    all the elements of A are also elements of B

$$A \subset B$$



  - "A is not a subset B": there exists at least one element of A which does not belong to B

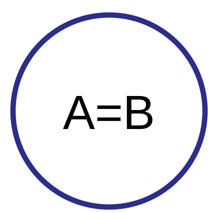
$$A \not\subset B$$



Equality of sets

- Two sets A y B are **equal** if both have the same elements.
  - The equality is equivalent to the **double inclusion**:

$$A = B \Leftrightarrow A \subset B \; and \; B \subset A$$



Set operations

- Given two sets A and B:

Union of A and B: set which is made up by the elements belonging to A, to B or to both sets.

$$\boxed{A \cup B = \{x \mid x \in A \; or \; x \in B\}}$$

Be careful with the usual language

Intersection of sets

Intersection of A and B: set which is made up by the elements belonging **simultaneously** to A and B.

$$\boxed{A \cap B = \{x \mid x \in A \; and \;\; x \in B\}}$$

More than two sets

- The union and the intersection of sets can be generalized to more than two sets:

$$A \cup B \cup C$$

$$A \cap B \cap C$$

- And they can be combined:

$$A \cap B \cup C$$

$$A \cup B \cap C$$

$$A \cap B \cup (C \cap D) \cup E$$

Difference of sets

Difference between A y B: set which is made up by the elements of A that do not belong to B.

$$\boxed{A-B=\{x \mid x\in A\ and\ x\notin B\}}$$

WARNING: A – B and B – A are different sets

Complement of a set

Let U be the universal set. Then, the **complement of A** is the set made up by the elements of U that do not belong to A.

$$\boxed{\overline{A} = \{x \in U \mid x \notin A\}}$$

It is fulfilled that: $\overline{A} = U - A$

Disjoint sets

- Two sets A and B are disjoint if their intersection is the empty set. That is, they haven't got common elements.

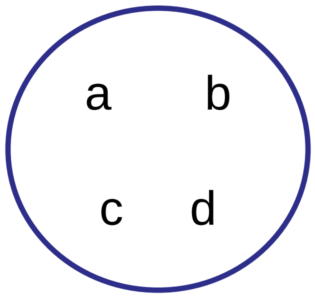
$$A \cap B = \phi$$

- Particular case: For any set *A*, *A* and its complement $\overline{A}$ are disjoint.

Cardinality of a set

- Given a set A, the cardinality or power of A is the number of elements belonging to A.

- Notation: $n(A)$



$$n(A)=4$$

Cardinality of the union of sets

- Given two sets A y B, the cardinality of the union of A y B is computed as

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

- $If\ A \cap B = \emptyset \rightarrow n(A \cup B) = n(A) + n(B)$

-Particular case:

$$If\ A \cap B = \emptyset \rightarrow n(A \cup B) = n(A) + n(B)$$

Key property to solve several problems

- Given <u>three sets</u> A, B y C, the cardinality of the union of the three sets is computed as

$$\boxed{\begin{aligned} n(A\cup B\cup C) = {} & n(A)+n(B)+n(C) \\ & -n(A\cap B)-n(A\cap C)-n(B\cap C) \\ & +n(A\cap B\cap C) \end{aligned}}$$

- Cardinality of the difference of sets:

$$\boxed{n(A-B)=n(A)-n(A\cap B)}$$

- Particular case:

$$If\ \ B\subset A\Rightarrow n(A-B)=n(A)-n(B)$$

- Cardinality of the complement set:

$$\boxed{n(\overline{A})=n(U)-n(A)}$$

Exercise 1

There are 30 students in a classroom. 15 of them play football and 20 of them play basketball. There are 10 students that play both sports.

1. How many of them play only football?

2. How many of them play any of the two sports?

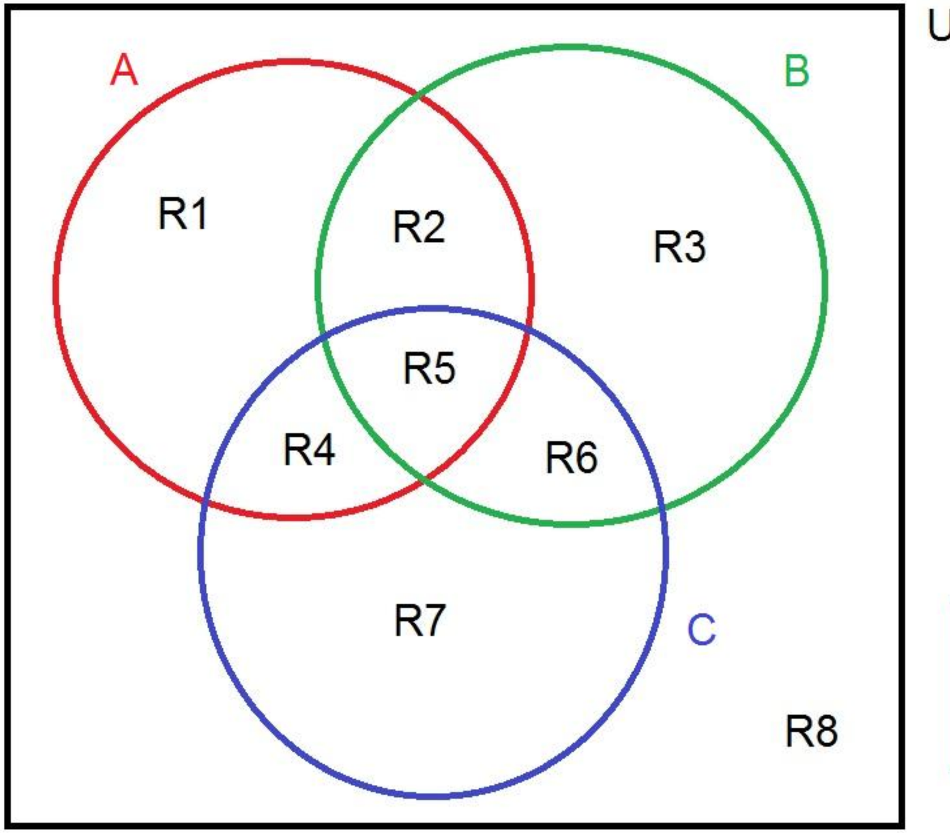
3. How many of them don’t play any of the two sports?

Exercise 2

From 60 students in a group, there are 38 who are registered in Mathematics, 30 in Plastic Arts and 40 in Geography. In addition, 22 of them are registered in both Mathematics and Plastic Arts, 27 in Mathematics and Geography, 20 in Plastic Arts and Geography, and 12 in the three subjects.

1. How many of them are registered in any of the three subjects?
2. How many are registered in Mathematics and Plastic Arts, but not in Geography?
3. How many are registered only in Mathematics?
4. And only in Plastic Arts?
5. And how many are not registered in any of the three subjects?
6. How many of them are registered in more than one subject?

Exercise 2



N(U)=60

DATA:

N(A)=38=R1+R2+R4+R5

N(B)=30=R2+R3+R5+R6

N(C)=40=R4+R5+R6+R7

N(A∩B)=22=R2+R5

N(A∩C)=27=R4+R5

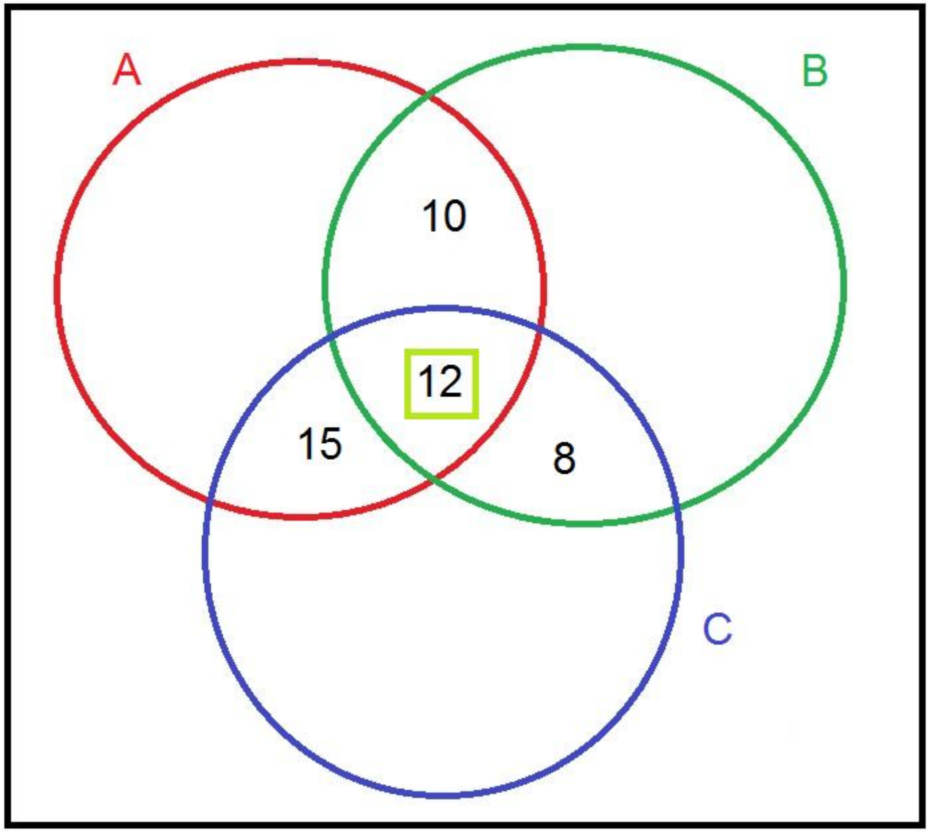
N(B∩C)=20=R5+R6

N(A∩B∩C)=12=R5

KEY DATUM

A={People registered in Mathematics}
B={People registered in Plastic Arts}
C={People registered in Geography}

Exercise 2



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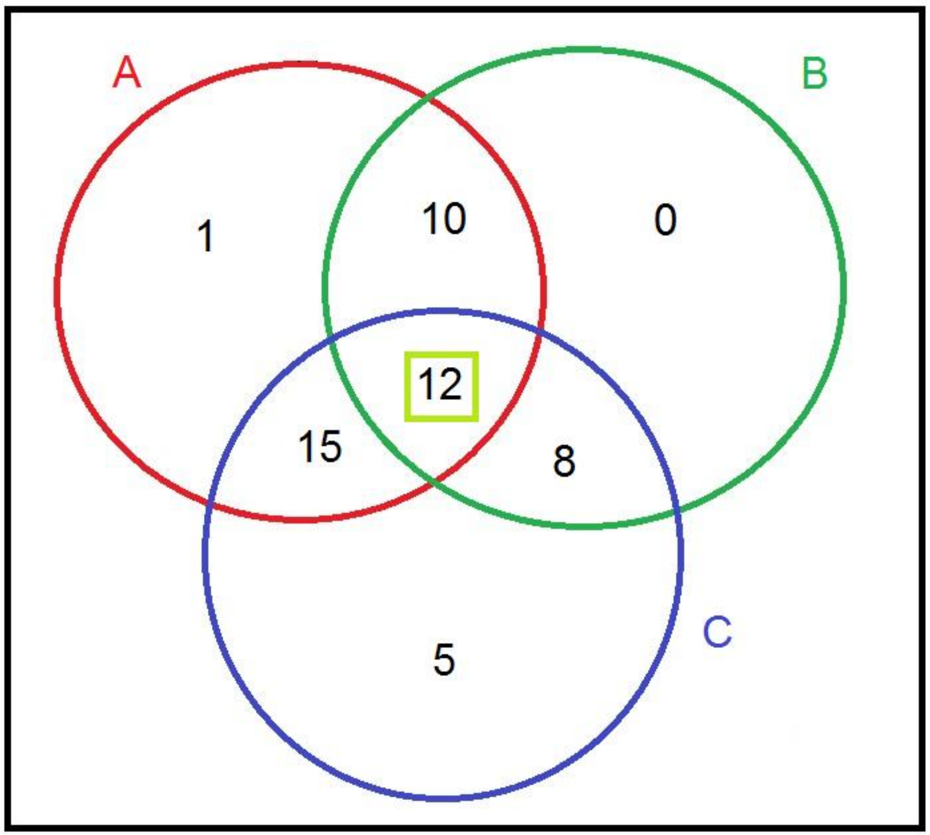

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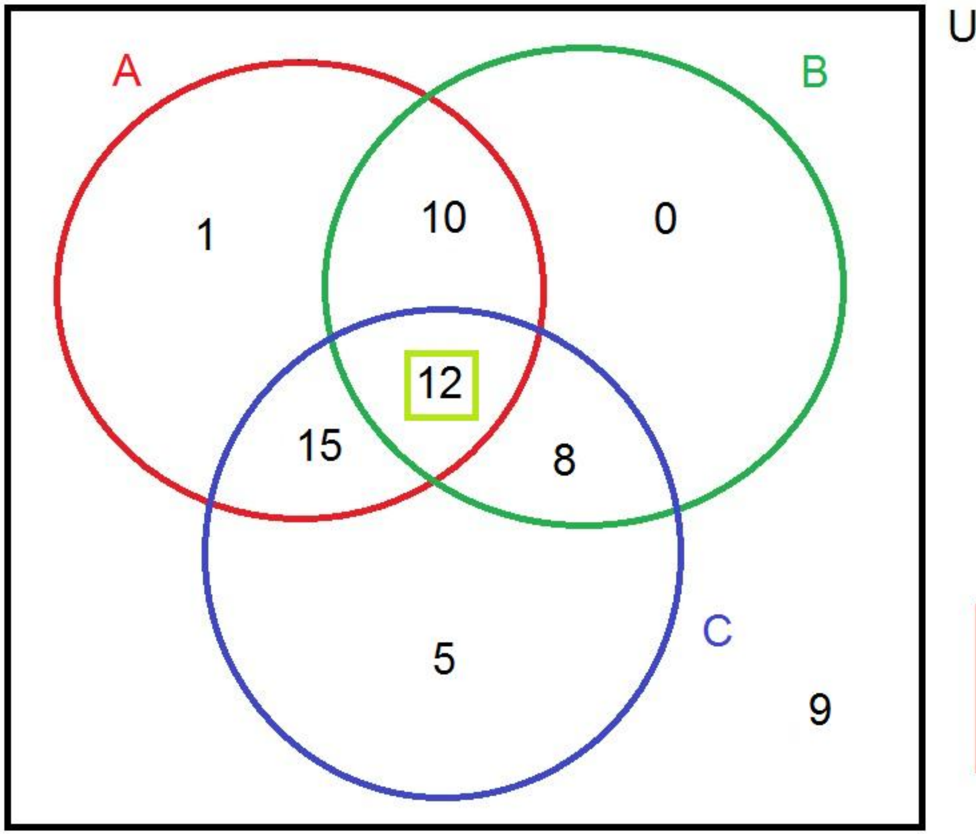

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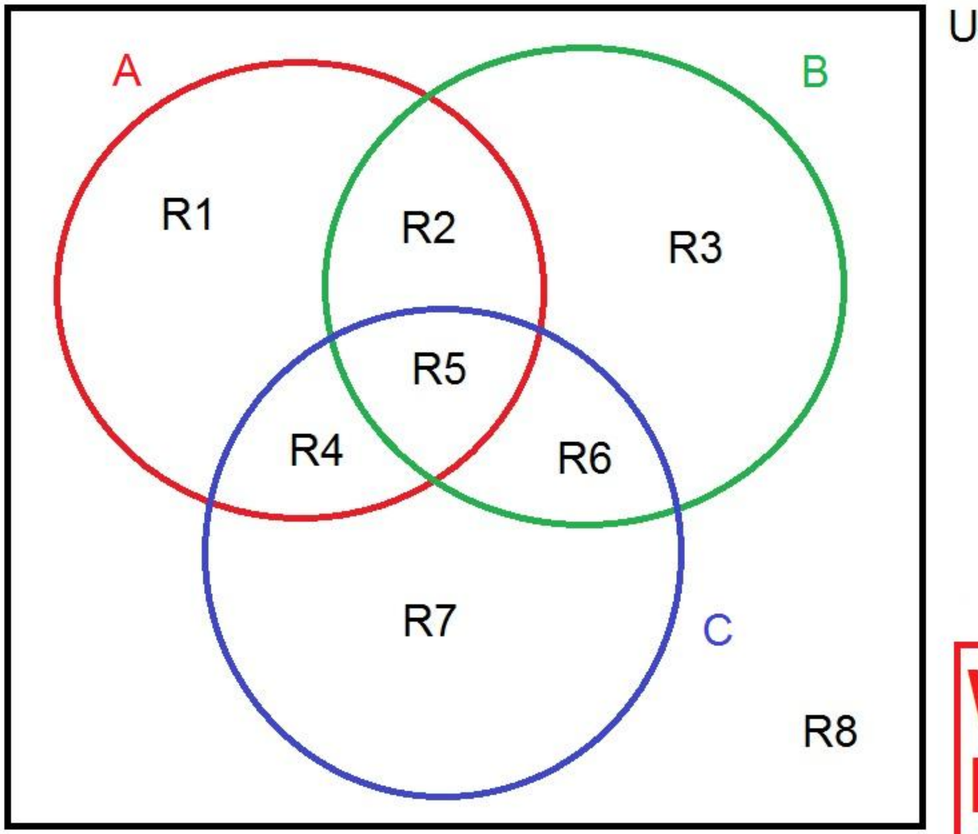

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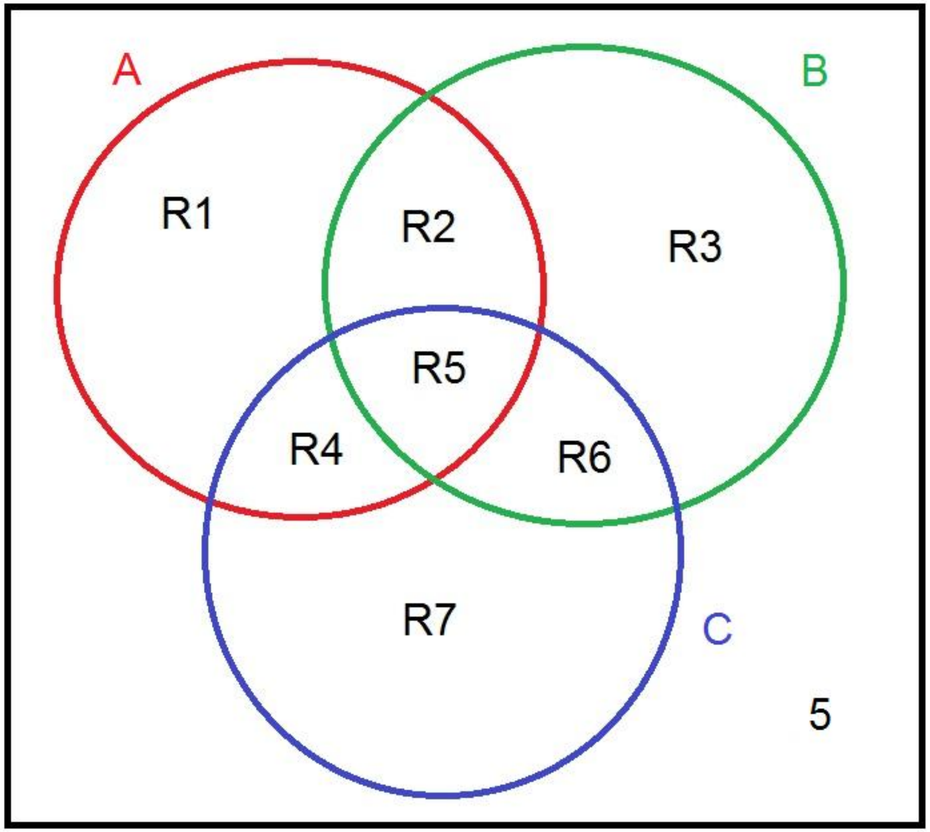
WHAT HAPPENS IF WE DON'T KNOW REGION 5?

Exercise 2



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Suppose that we know how many people is not registered in any of the three subjects =R8

$\boxed{\text{For instance } R8=5}$

$\Downarrow$

$\boxed{N(A \cup B \cup C)=60-5=55}$

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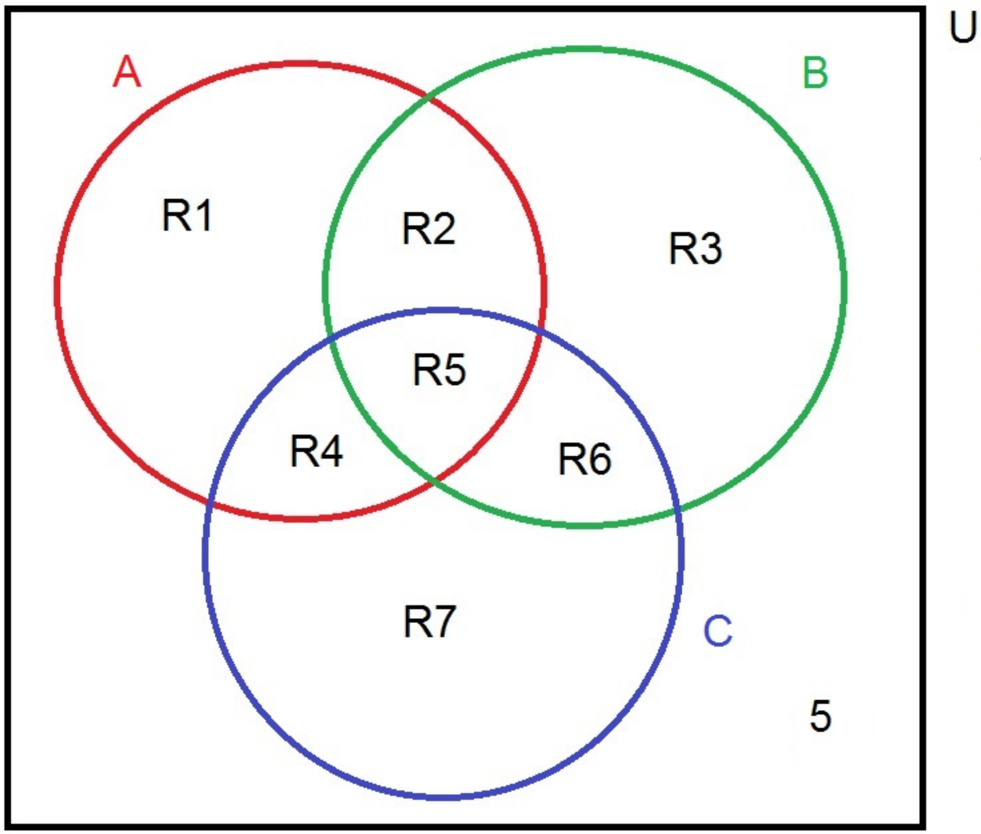
Exercise 2



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$$\mathbf{N(A\cup B\cup C)=R1+R2+R3+R4+R5+R6+R7=55}$$

$$\mathbf{N(A)+N(B)+N(C)=(R1+\boxed{R2}+\boxed{R4}+\boxed{R5})+(\boxed{R2}+R3+\boxed{R5}+\boxed{R6})+(\boxed{R4}+\boxed{R5}+\boxed{R6}+R7)}$$

$$\mathbf{N(A)+N(B)+N(C)-N(A\cap B)-N(A\cap C)-N(B\cap C)=R1+R2+R3+R4+R6+R7}$$

$$\mathbf{N(A)+N(B)+N(C)-N(A\cap B)-N(A\cap C)-N(B\cap C)+N(A\cap B\cap C)=R1+R2+R3+R4+R5+R6+R7}$$

$$\Rightarrow \boxed{\mathbf{N(A\cup B\cup C)=\ N(A)+N(B)+N(C)-N(A\cap B)-N(A\cap C)-N(B\cap C)+N(A\cap B\cap C)}}$$