



Unit 8: Introduction to interval estimation

Class exercises

WATER Children's weekly water consumption (X , in litres) is a variable that is normally distributed with dispersion $\sigma = 3.5$. In a simple random sample of 45 children, the weekly average consumption of water is 17.72 litres.

- a) Obtain a confidence interval (90%) for the expected consumption.
- b) If water consumption is not normally distributed, what happens with the confidence interval?
- c) Determine the minimum sample size to estimate the expected consumption with a margin of error of 1 litre and a confidence level of 95%.

INCOME Monthly salary (X , in euros) has been analyzed over a random sample of 46 employees in a multinational company, obtaining:

$$\bar{x} = 1360.65; s = 93.36$$

- a) Assuming that salaries are normally distributed, obtain a confidence interval for the expected salary with a confidence level of 90%.
- b) If confidence level is increased, what happens with interval precision?
- c) If salaries are not normally distributed, what happens with the confidence interval?

ECOLOGY An environmental organization is analyzing wastewater in rivers. In particular, the organization is interested in the fee paid by the companies responsible for such wastewater (in €), a variable that follows an unknown distribution with standard deviation 5,000.

- a) Determine the minimum sample size to estimate the expected value of the fee with a confidence level of 90% and a margin of error of 1,000 €.
- b) What happens with size if margin of error reduces to a half?

ELECTIONS A political party is analyzing its electoral prospects and financial conditions.

- a) In a random sample of 60 individuals, 24 of them said they would vote for the party. Obtain a confidence interval for the proportion of voters with a confidence level of 95%.
- b) Determine the minimum sample size to estimate the proportion of voters of the party with a confidence level of 95% and a margin of error of 0.15.
- c) Those in charge of party's finances have observed that their incomes are very homogeneous, but they are unaware of the level of dispersion of expenditure. Given that the annual expenditure is normally distributed and that, in a random sample of 29 years the variance of "Expenditure" (in hundreds of thousands of



euros) is $s^2 = 8898.608$, obtain a confidence interval for the variance with a confidence level of 90%.

d) If confidence level is 99%, what happens with the above confidence interval?.

[Adapted from *Economic Data Analysis II. Inferential methods*, problem 7.2, p. 374-375]

BANK A bank's customer service is analyzing customer's waiting times (in minutes) in a certain bank branch, a variable that is normally distributed.

Given a random sample of 46 clients:

- Obtain a confidence interval for the variance with a confidence level of 99%, knowing that sample variance was 36.
- Obtain a confidence interval for the proportion of customers waiting more than 10 minutes with a confidence level of 90%, knowing that in the previous sample 16 customers waited more than 10 minutes.

DRAWING On a random sample of 15 students in a drawing course, the average score was 7.5.

- Given that students' scores follow a normal distribution with standard deviation 1, obtain the expected score of the class with a confidence level of 95%.
- In another class, a random sample of 22 students showed an average score of 6. If these scores are also normally distributed with standard deviation 1.6, obtain a confidence interval for the difference between the expected scores with a confidence level of 90%.



Proposed exercises

TRANSPORT The manager of a certain carrier is studying daily income, a variable that is normally distributed. A random sample of 8 days reveals an average daily income of 525 euros with standard deviation 116.5.

- a) Estimate the expected daily income with a confidence level of 95%.
- b) Previous studies show that the standard deviation of income is 150 euros:
 - b1) How would this information affect the result of section a)?
 - b2) Determine the minimum sample size to reduce the interval width in b1) to a third, keeping the same level of confidence.

Results

a) (427.51, 622.49) **b1)** (421.025, 628.975) **b2)** $n = 72$

BUTANE The annual household spending on butane (X, in euros) is a normal random variable with known standard deviation. In order to estimate the expected annual expenditure with certain guarantees of accuracy and confidence it would be necessary to investigate a sample of 200 families.

Justify what happens with sample size if, keeping the same level of confidence, we were to double the width of the interval.

Results

50

CONSUMPTION The annual electricity household consumption is a random variable (X, in kWh) whose distribution is unknown. A random sample of 23 households shows:

$$\bar{x} = 1800; s = 341.54$$

- a) Obtain a confidence interval for the expected annual consumption with a confidence level of 95%.
- b) If electricity consumption were normally distributed, how would this affect the above interval?
- c) Obtain a confidence interval if the population is normally distributed with standard deviation $\sigma = 340$.
- d) If the previous interval was to be reduced to a half, how much do you need to increase sample size, keeping the same level of confidence?

[Adapted from *Economic Data Analysis II. Inferential methods*, problem 7.4, p. 377].

Results

a) (1481.66, 2118.34). **b)** (1652.31, 1947.69). **c)** **(1661.05, 1938.95)**. **d)** $n = 92$



DEVELOPMENT The R&D funds (X , in thousands of euros) of a multinational company is a normally distributed variable with dispersion $\sigma = 3$. A random sample of 35 subsidiaries belonging to the corporation shows $\bar{x} = 24.29$.

- Obtain a confidence interval for the expected R&D funds with a confidence level of 90%.
- Out of the 35 subsidiaries, only 65.71% have public-funded R&D projects. Obtain a confidence interval for the proportion of public-funded R&D projects with a confidence level of 95%.

Results

a) (23.451; 25.120). **b)** (0.498; 0.816).

THEATRE The director of a theater is analyzing box-office monthly income. In order to do this, he analyzes a random sample of 29 months and obtains: $\bar{x} = 1202.15$; $s = 102.39$

- Estimate the expected monthly income with a confidence level of 90%.
- A random sample of 100 tickets shows that 20 of them have some discount. Calculate an interval for the proportion of tickets sold with discount with a confidence level of 95%.

Results

a) (1142. 02; 1262. 28). **b)** (0.121; 0.278).

MILK In order to study household monthly consumption of milk (in litres), we analyze a random sample of 25 households, obtaining an average consumption of 54 litres with standard deviation 3.3.

Given that milk consumption is normally distributed:

- Obtain a confidence interval for the expected consumption of milk with a confidence level of 95%.
- Estimate the variance of consumption with a confidence level of 90%.

Results

a) (52.6 38 55.3 62). **b)** (7.18;18.8 7).



REPAIRS The annual expenditure in repairs (in thousands of euros) in a car rental company is a random variable that follows a normal distribution. A random sample of 56 cars shows an average expenditure of 3 with standard deviation 0.9.

- a) Estimate the expected annual expenditure with a confidence level of 95%.
- b) Estimate the variance of the annual expenditure with a confidence level of 90%.

Results

a) (2.76, 3.24) **b)** (0.608, 1.144)

INTERNET An Internet services provider company is carrying out a study on household Internet penetration.

What sample size is needed in order to estimate the proportion of households with Internet access with a confidence level of 99% and a margin of error of 0.10?

Results

$n = 166$

HEALTH EXPENDITURE Household monthly health spending (X , in euros) is a normally distributed random variable with standard deviation 4.69.

- a) A random sample of 51 households shows an average spending of 43.57 euros. Obtain a confidence interval for the expected spending with a confidence level of 90%.
- b) There has been a public campaign aimed at cutting down health care spending. Spending after the campaign is a random variable that follows a normal distribution with standard deviation of 5.20. Given that, after the campaign, a new random sample of 27 households reveals an average spending of 38.46 euros. Estimate the expected reduction in health spending with a confidence level of 95%.

Results

a) (42.49; 44.65). **b)** (2.76; 7.46).