Unit 4: Continuous Probability Models

A. Class exercises

CALL **The duration of calls from users of a new telephone company is randomly distributed between 10 and 60 minutes.**

a) **Calculate the probability that the duration of a call exceeds 45 minutes.**
b) **What is the probability that duration of call is between 25 and 35 minutes?**
c) **What is the expected duration? And its variance?**
d) **The company is launching a promotional campaign that includes a discount coupon for each call lasting longer than 45 minutes. If a customer makes 10 calls in day, what is the probability that he/she gets at least one discount coupon?**
e) **What is the expected number of calls a customer needs to do in order to get a discount? Calculate the probability that the sixth call is the first one to get a discount.**

TABLES **Given a random variable Z that follows a standard normal distribution, calculate the following probabilities and values:**

a) **P (Z ≤ 1.65)**
b) **P (Z ≤ -1.65)**
c) **P (-1.65 <Z <0.61)**
d) **P (|Z| ≤ 1.96)**
e) **P (|Z| > 1.96)**
f) **What is the eighth decile of the distribution? And the first decile?**

CANS **The weight of tinned beans (in grams) is a random variable with mean 150 and standard deviation 12.**

a. **Calculate the probability that a tin of beans weighs more than 160 grams.**
b. **Obtain the probability that the weight deviates, from the expected value, a maximum of 10 grams.**
c. **What is the maximum weight of the 90% of tins?**
d. **If the factory cost of each tinned gram of beans is 0.05 euros, obtain the probability that the total cost of a tin is lower than 8 euros.**
e. **If 10 cans are randomly selected, what is the probability that any of them weighs more than 160 grams of product?**

B. Proposed Exercises

TEMPERATURES **A travel agency is offering a vacation in a city where the expected temperature (°C) is $\mu = 25$ with variance $\sigma^2 = 75$. Three hypotheses are considered about the distribution of temperatures: unknown, uniform (between 10 and 40 °C) or normal distributions. Calculate the probability that the temperature of any day is in the range of 1.2 standard deviations with respect to μ in the three previous situations.**

RESULTS

a) Unknown distribution: ≥ 0.3055
b) Uniform distribution: 0.6928
c) Normality distribution : 0.7698

Greengrocer **The weight of the packed trays of apples sold in a supermarket is randomly distributed between 1.420 and 1.540 kg.**

a) **If a tray is randomly selected, what is the probability it weighs less than 1.5 kg? What is the expected weight?**
b) **The manager of the grocery store receives a commission of 0.5 euros for every customer purchasing a tray of apples of less than 1.5 kg. If 50 customers come to the supermarket, what is the probability that he/she obtains a commission of at least 20 euros?**
c) **If 2 out of 10 customers buy at the grocery, calculate the probability that on a day the first customer to buy in the grocery is the fifth who comes to the supermarket. What is the expected value?**

RESULTS

a) 0.667; 1.48.
b) 0.132
c) 0.0819; 5.

Water **The monthly water consumption of households in a country (in $m^{3)}$ follows a normal distribution N(15,3).**

a) **Calculate the probability that the monthly consumption of a household is over 19 m^3.**
b) **Complete the following statement: The monthly water consumption of the 25% of households is below _____ $m^{3.}$**
c) **Among those households that consume more than 19 m^3 per month, 15% of them waste water; this rate is 8% among households which monthly consumption does not exceed that amount. What is the probability that a randomly selected household waste water?**

RESULTS

a) 0.0918
b) 12.99
c) 0.0864

SCORES **The scores got by students on a final exam follow a normal distribution with mean 72 and variance 9.**

a) **The top 10% of the students got an A and the bottom 25% failed. Calculate the corresponding minimum and maximum scores.**
b) **Considering 15 randomly selected students, what is the probability that more than one has failed the exam?**

[Adapted from PERALTA, MJ and others (2000): *Estadística. Problemas resueltos*.Ed. Pirámide. Exercise 15, p.. 376]

RESULTS

a) 75.84; 69.99.
b) 0.9198

TAPES **The duration of some video tapes (in minutes) follows a normal distribution with μ = 180 and σ = 12.**

a) **Calculate the ratio of tapes which duration deviates, from the expected value, a maximum of a quarter of an hour.**
b) **If someone uses the tape to record a 2-hour fil, what is the probability that he/she loses the end because of space?**
c) **If someone wants to record a tennis game, what is the maximum length of the game in order to guarantee it has been fully recorded with a probability of 10%?**
d) **Those tapes which last less than 165 minutes can be returned to the manufacturer. If a customer bought 6, calculate the probability that any of them is returned.**

RESULTS

a) 0.788
b) 0.62
c) 195. 38
d) 0.4881