Unit 1: Uncertainty and probability

A. Class exercises

COSTUMES **A group of friends have organized a costume party attended by a total of 12 people.**

a. **The evening begins with a draw of 4 different gifts. How many ways can the gifts be awarded? What if it is previously agreed that each person cannot be awarded more than once?**
b. **The organizers have rented a total of 15 costumes. How many different ways can the friends dress up? What if they had only 12 dresses available?**

[From Análisis de Datos Económicos II. Métodos Inferenciales, problem 1.2 pg. 51-52]

SURVEY **A survey on environmental protection is being conducted in a town where 20 companies are located. It has been decided to randomly select 5 different companies to answer the survey.**

a. **How many different ways may companies be selected to participate in the survey?**
b. **If 12 of the companies in the town are small or medium-sized and 8 are big, what is the probability that two large companies are included in the sample?**

EQUIPMENT **A survey on household equipment in a given city has obtained that the probability of having a home computer is 0.72, while the probabilities of having a digital camera or a DVD player are, respectively, 0.61 and 0.92. Assuming that these events are independent:**

a. **Calculate the probability that a household has a digital camera and a DVD player.**
b. **What is the probability that a household has a digital camera or a DVD player?**
c. **What is the probability of having a digital camera but not a computer?**
d. **What is the probability of having a digital camera for a household having a DVD player?**

BASIC PROPERTIES **Demonstrate that as a consequence of the axioms of Kolmogorov, given any two events A and B, the following properties hold:**

1. $P(A^c) = 1 - P(A)$
2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

LOTS **We have two lots with pieces from the same production process. The first batch ($L_{1)}$ has 1% defective units, while the second one (L_2) has 5%. The first batch of parts is twice as big as the second one. A piece is randomly selected from these lots:**

a. **Calculate the probability that the chosen piece is defective.**
b. **If the piece is defective, what is the probability that it comes from the second batch?**

[From MARTÍN-PLIEGO, F.J y RUIZ-MAYA, L. (2006): *Fundamentos de Probabilidad,* Editorial Thomson]

B. Proposed exercises

EDUCATION **A group of students from the Faculty of Economics and Business Administration is conducting a study on the educational level of the population in Asturias. They have used the data generated by SADEI from 2001 Population Census [http://www.sadei.es/] collected in the following table:**

EDUCATION LEVEL	MEN (M)	WOMEN (W)
Illiterate (I)	2726	5186
No education (N)	40327	61075
First grade (F)	106909	131369
Second grade (S)	230163	217491
Third grade (T)	53952	64061
Unknown (U)	11562	12877
TOTAL	445639	492059

a. **What is the probability that a randomly selected person is a woman with third grade education? What kind of probability are we using?**
b. **Calculate the probability that a person selected at random has third grade education.**
c. **Given a person with second-degree studies, calculate the probability that she is a woman.**

Results. (a) $P(M \cap T) = 0.068$. (b) P(T)=0.1259. (c) $P(M / S) = 0.486$.

BONUS **A large company consults its employees on a new bonus scheme. The plan is supported (S) by 65% of all night-shift workers (N) and 40% of women (W). In addition, 50% of employees belong to the night shift and 30% of all employees are women. Finally, 20% of the night shift workers are women.**

a. **What is the probability that a randomly chosen employee is a woman and supports the plan?**
b. **What is the probability that a randomly chosen employee is a woman and / or a night shift worker?**
c. **Are the sex and the shift of the employees independent?**
d. **What is the probability that a person is in the night shift, knowing that she is a woman?**

Results. (a) $P(S \cap W) = 0.12$. (b) $P(W \cup N) = 0.7$. (c) $P(W \cap N) \neq P(W)P(N)$. (d) $P(N / W) = 0.333$.

REGISTRATION **Justify how many cars can be registered with the current system (4 digits followed by 3 letters of a total of 21, excluding vowels and some consonants, appear on each plate).**

Results. Total No. = $VR_{10,4}\ VR_{21,3}$=92,610,000 (92.61 million).

DEPARTMENT **The sales department of a company has a team of 11 members. The company has 4 tickets for a concert, to be distributed among the members of the team.**

a. **How many ways can these 4 tickets be assigned to the 11 members of the department?**
b. **If 8 of the members are directors and 3 are engineers, calculate the probability that three executives are awarded.**

Results. (a) $C_{11,4} = \frac{11!}{7!4!} = 330$. (b) $p = \frac{3(56)}{330} = 0.509$

LOTERIA PRIMITIVA **Calculate the probability of winning the following bets in the *lotería primitiva*.**

a. **Make full (six hits).**
b. **Matching three numbers.**

(Note: The *lotería primitiva* is a lottery where 6 numbers are randomly drawn without replacement from a total of 49 numbers).

Results. (a) $P(full) = \frac{1}{13983816} = 7.15 \cdot 10^{-8}$. (b) $P(three) = \frac{246820}{13983816} = 0.0176$.

FISHERMEN **In a coastal town 40% of fishermen are young (Y). In this group only 5% are captains (C) of the boat in which they work, while among adults this percentage rises to 20%.**

a. **Calculate the probability that a randomly selected fisherman is the captain of his boat.**
b. **If you know a fisherman works as a captain, what is the probability that he is young?**

Results. (a) $P(C) = 0.14$. (b) $P(Y/C) = 0.143$.

CAMERA **The chamber of commerce in a town has 54 member companies. A delegation of 6 companies is to be sent to an exposition.**

a. **How many different ways can you organize the delegation?**
b. **80% of the companies associated with the chamber of commerce are small (S). In this group 8% of companies buy products on the Internet (I), whereas this proportion increases to 16% among all the other (i.e., large) firms. Obtain the probability that a randomly selected company is small if we know that it makes purchases online.**

Results. **(a)** 25,827,165. (b) P(S/I) = 0.67.